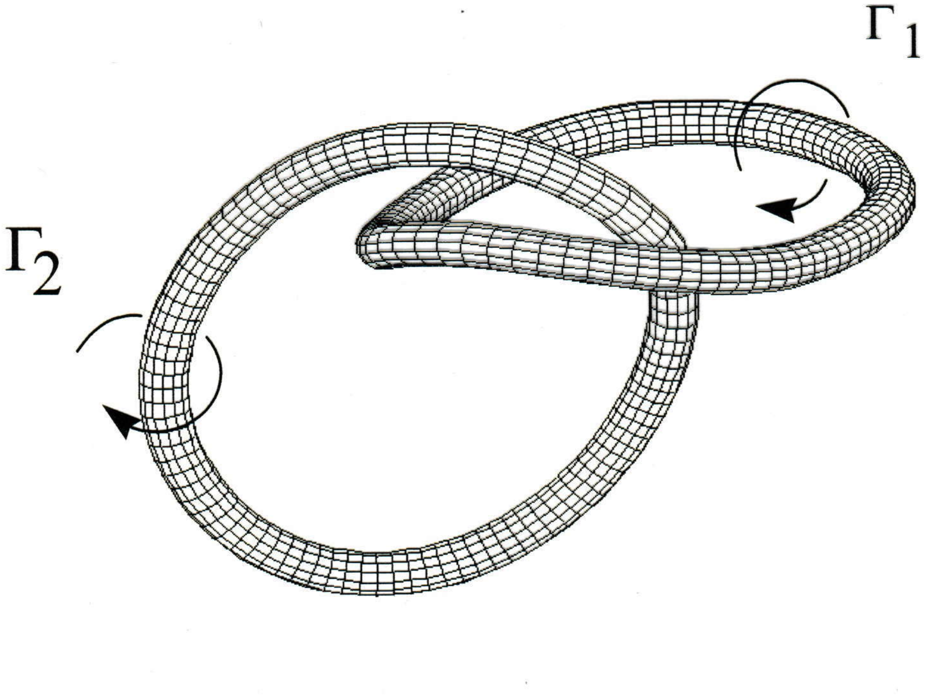
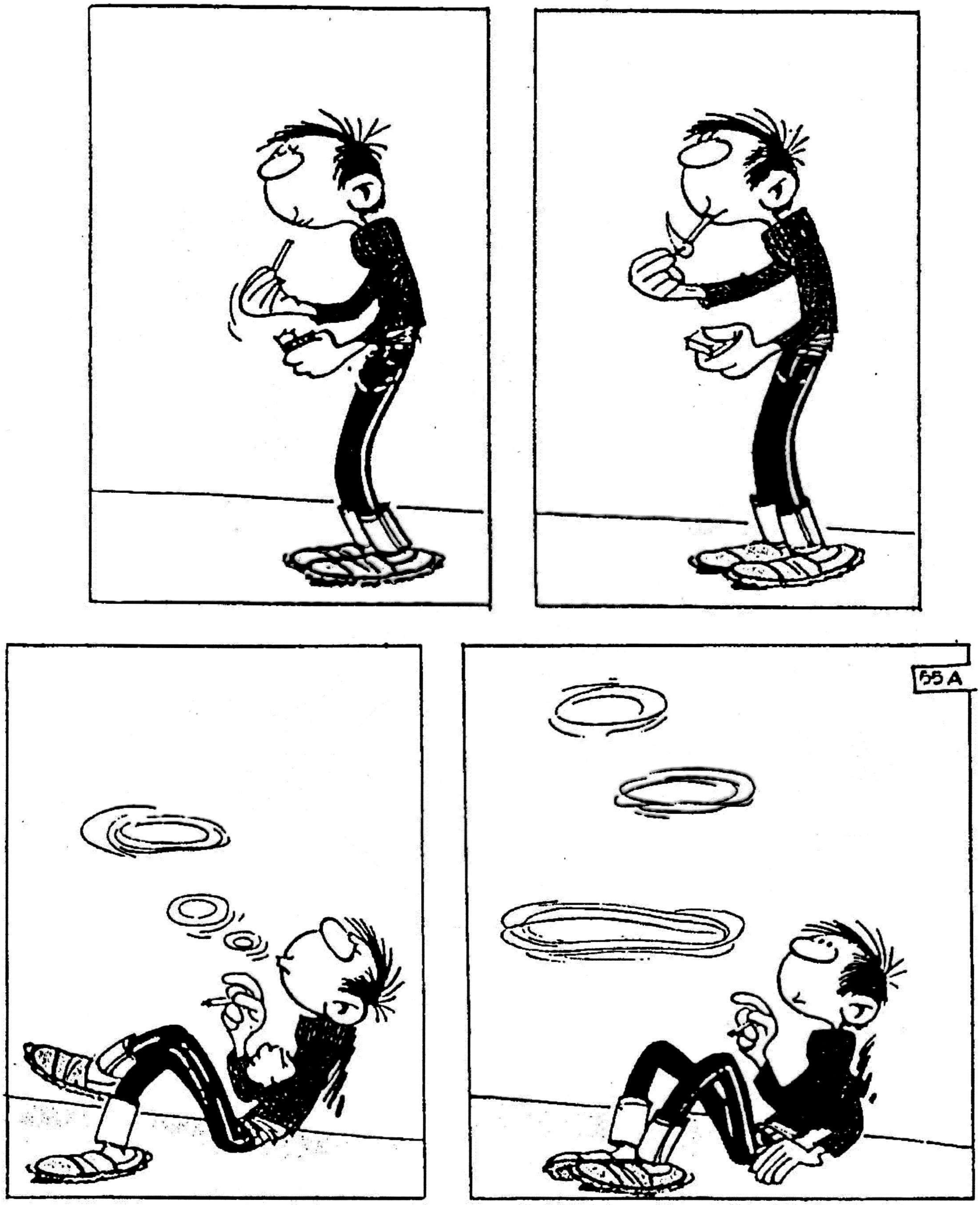
Mouvement et Dynamique des Filaments et des Anneaux Tourbillons de Faible Epaisseur

Daniel Margerit

6 novembre 97

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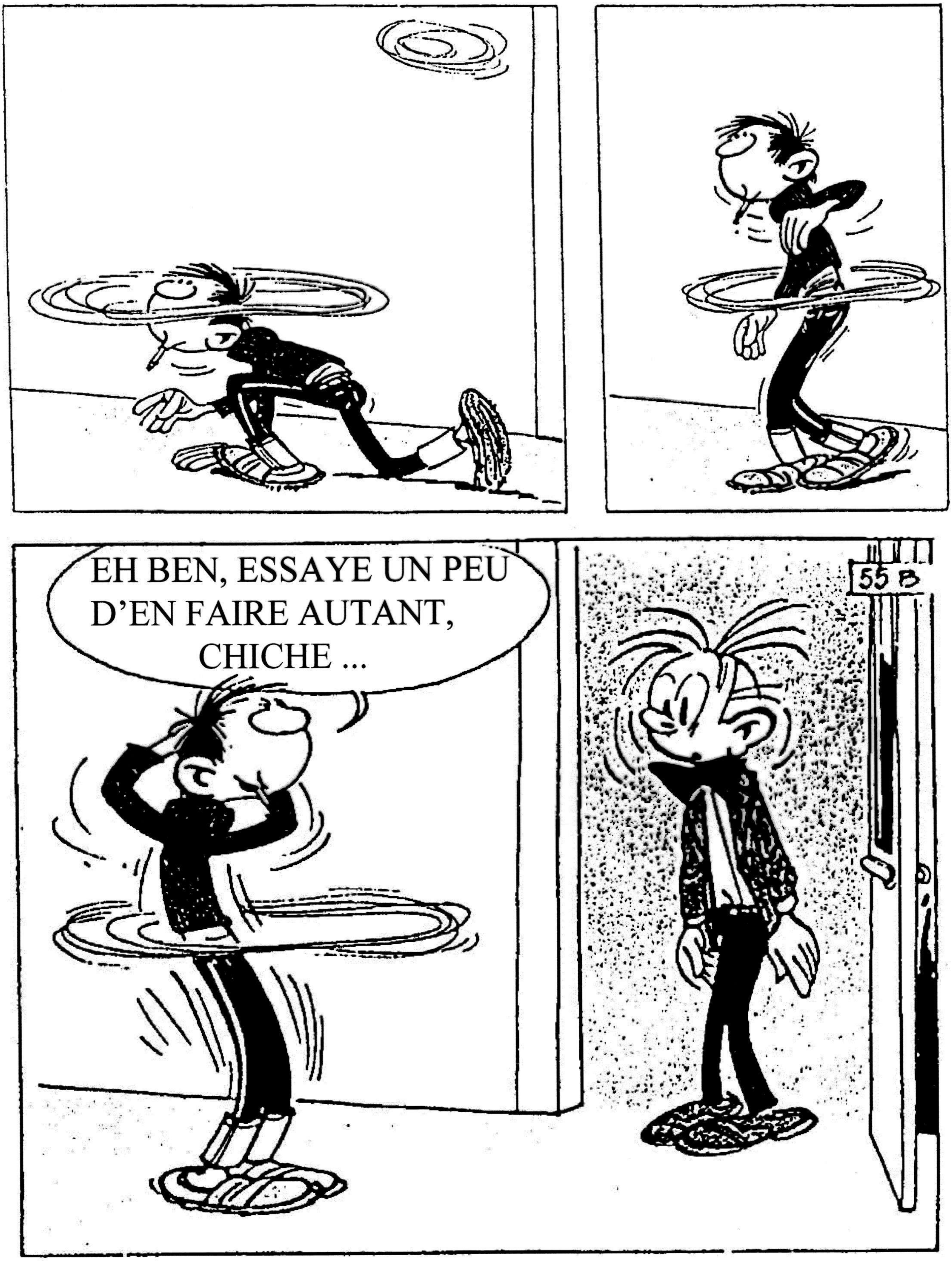
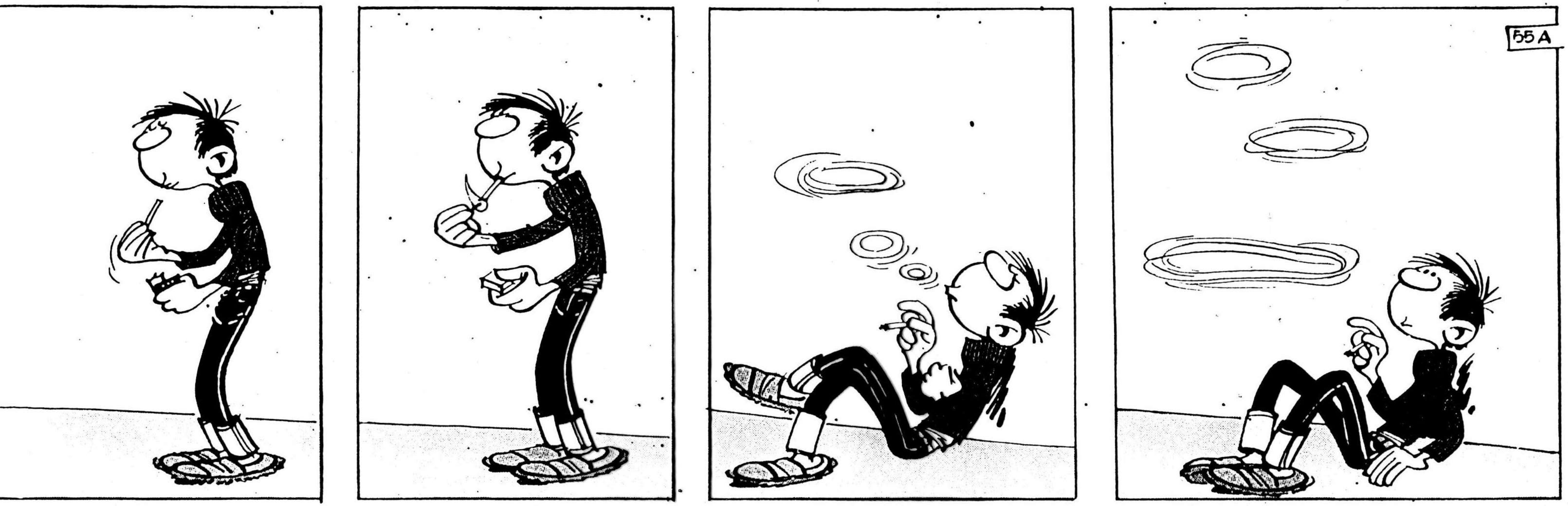

EH BEN, ESSAYE UN PEU D'EN FAIRE AUTANT, CHICHE ...
55 B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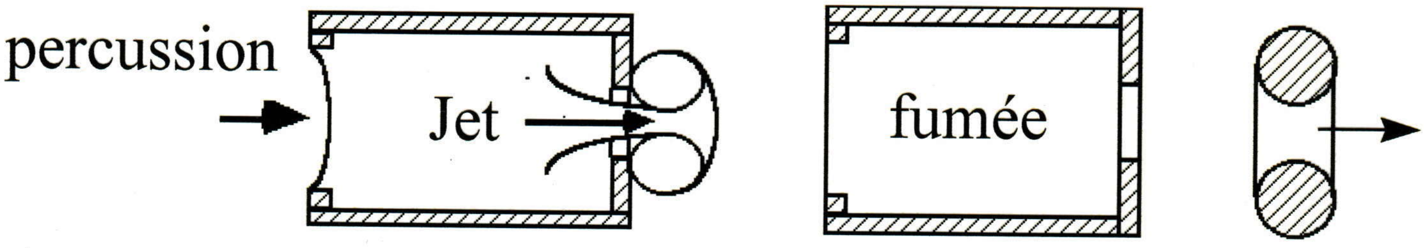
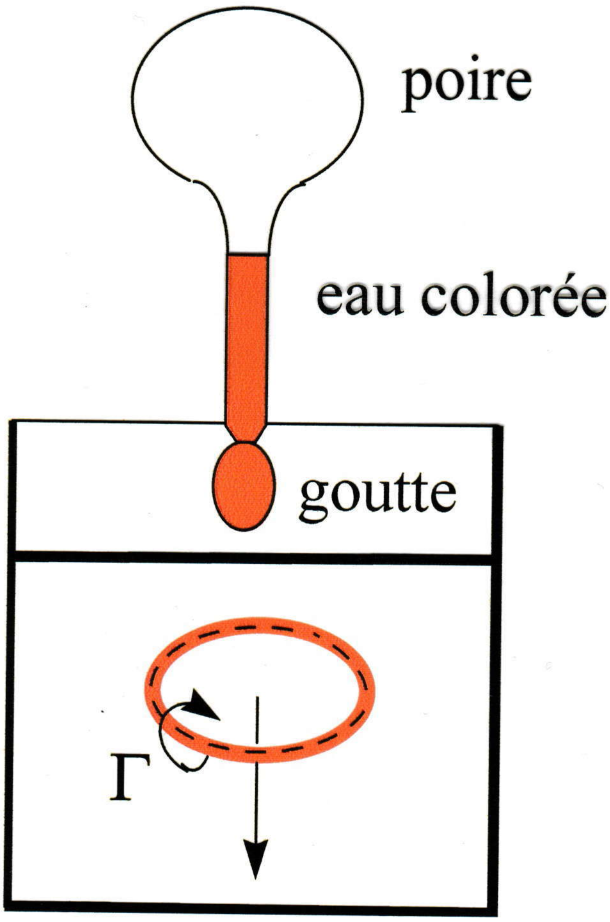
EH BEN, ESSAYE UN PEU D'EN FAIRE AUTANT, CHICHE...


percussion
Jet
fumée


poire
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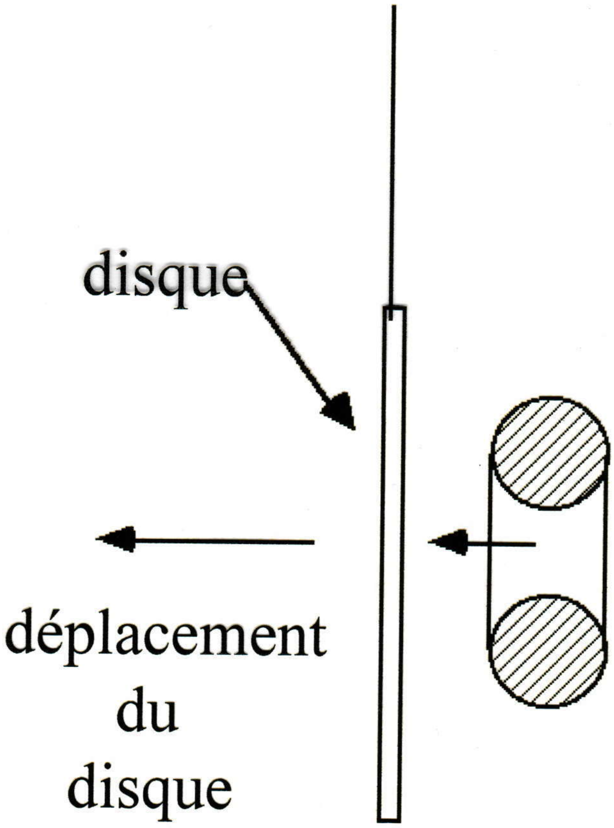
disque
déplacement
du
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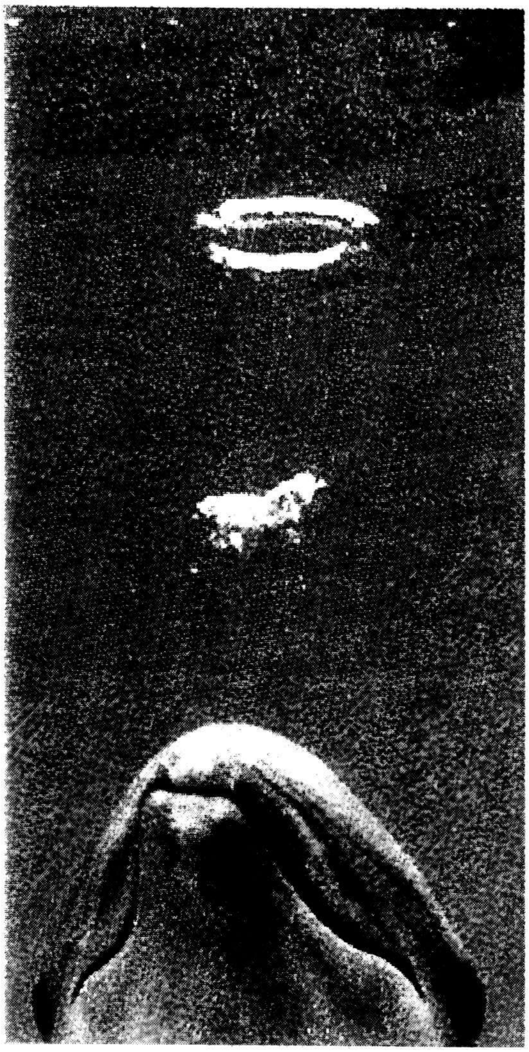


Figure 12 : Bulle torique faite par un dauphin

a b c

Figure 4 : Photographies de tornades

• Domaine d'étude :

Ecoulements rotationnels laminaires incompressibles et Re>>1

$$\frac{\partial \vec{\mathbf{v}}}{\partial t} + \vec{\mathbf{v}}\, grad\, \vec{\mathbf{v}} = -grad\, \frac{p}{\rho} + \nu \Delta \vec{\mathbf{v}}$$

$$div\, \vec{\mathbf{v}} = 0$$

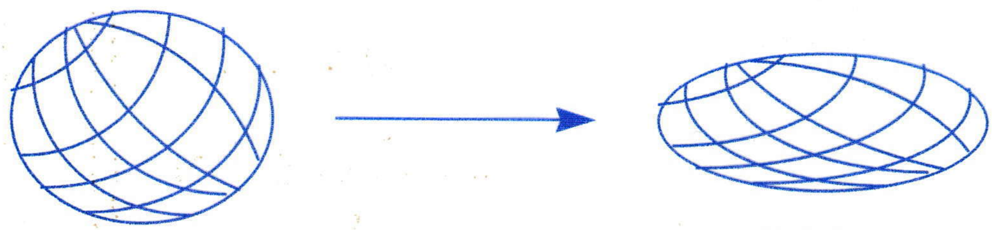
• Le champ de Vorticité : $\vec{\omega} = rot(\vec{\mathbf{v}})$

$$\frac{\partial \vec{\omega}}{\partial t} + \vec{\mathbf{v}}\, grad\, \vec{\omega} = \vec{\omega}\, grad\, \vec{\mathbf{v}} + \nu \Delta \vec{\omega}$$

$$\vec{\mathbf{v}}(\vec{\mathbf{x}}) = \overrightarrow{grad}(\varphi(\vec{\mathbf{x}})) \quad + \quad \frac{1}{4\pi} \iiint \frac{\vec{\omega}' \wedge (\vec{\mathbf{x}} - \vec{\mathbf{x}}')}{\left|\vec{\mathbf{x}} - \vec{\mathbf{x}}'\right|^3} dx'$$

vitesse potentielle + vitesse induite

$\Delta \varphi = 0$ CL et CI



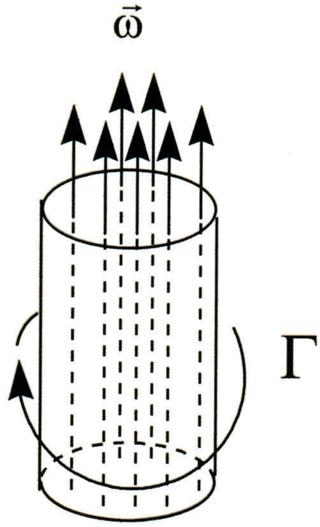
$\Omega(t = 0)$ $\Omega(t)$?

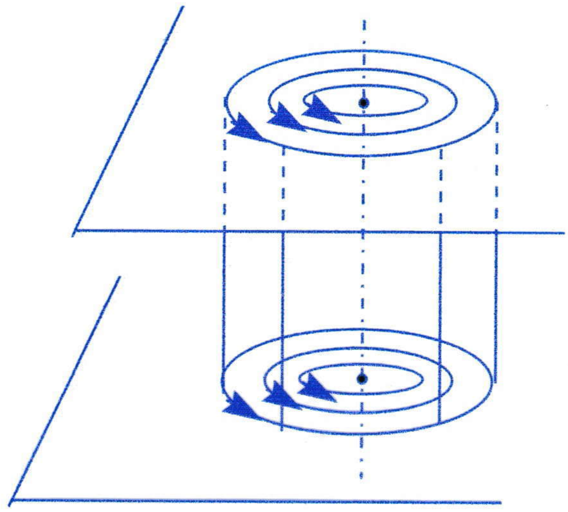
$\vec{\omega}(\vec{\mathbf{x}}, t = 0)$ $\vec{\omega}(\vec{\mathbf{x}}, t)$?

$\vec{\mathbf{v}}(\vec{\mathbf{x}}, t = 0)$? $\vec{\mathbf{v}}(\vec{\mathbf{x}}, t)$?

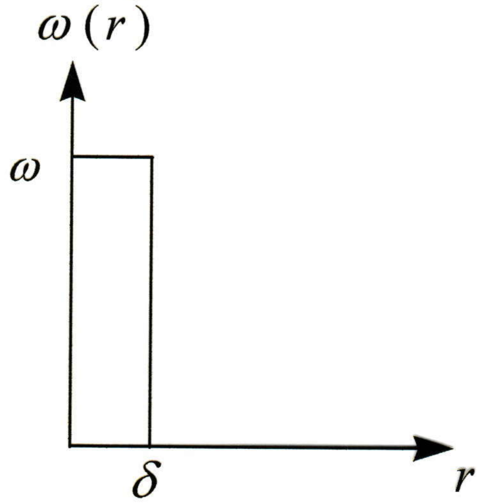
Vorticité | gelée concentrée

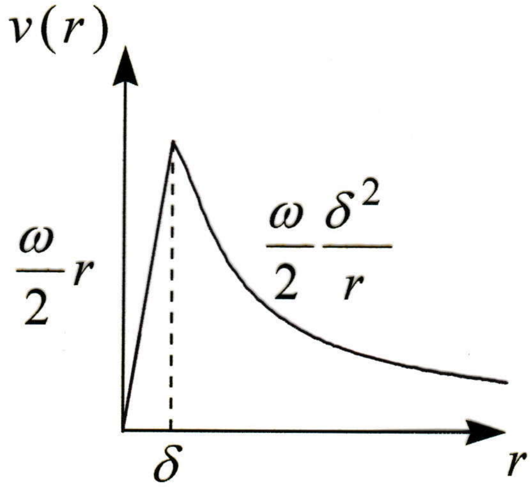
- # Une Tornade :



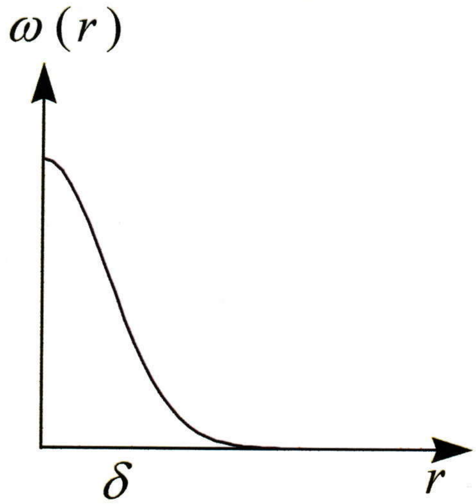


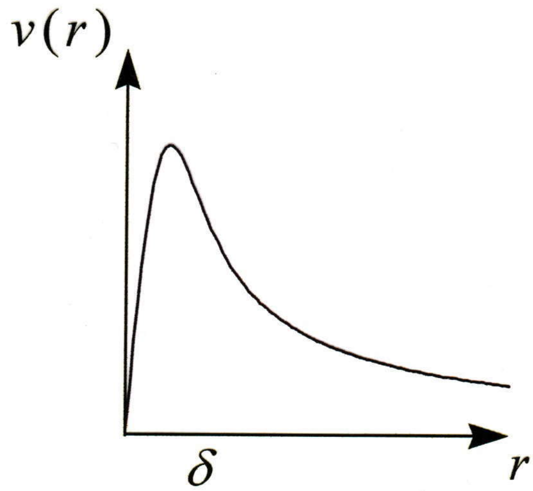
non visqueux : Rankine





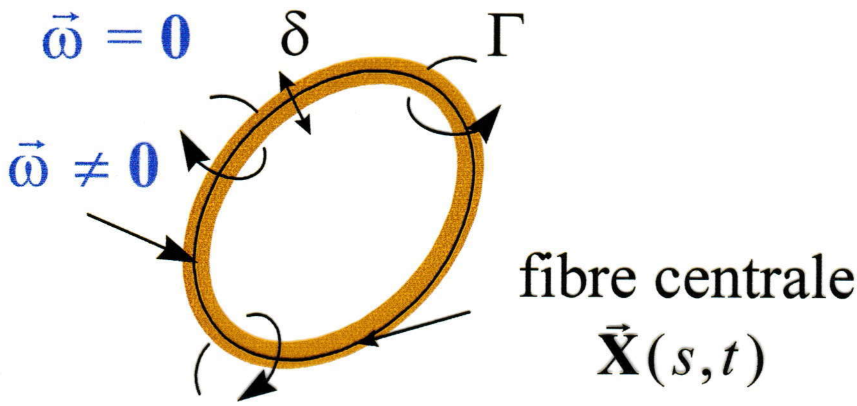
visqueux : Burger



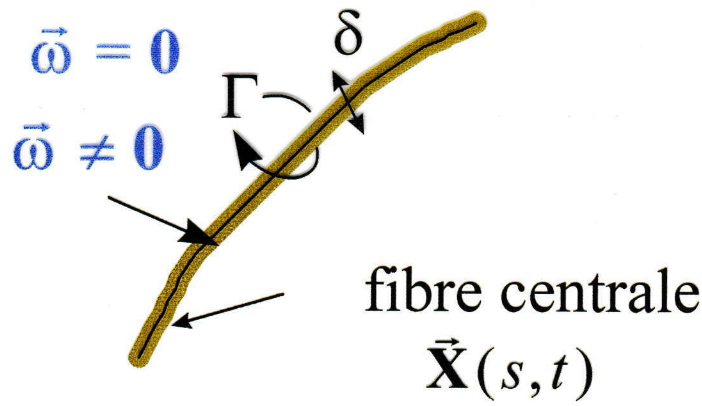


- Anneaux et filaments de faibles épaisseur :

Anneau tourbillon



Filament tourbillon



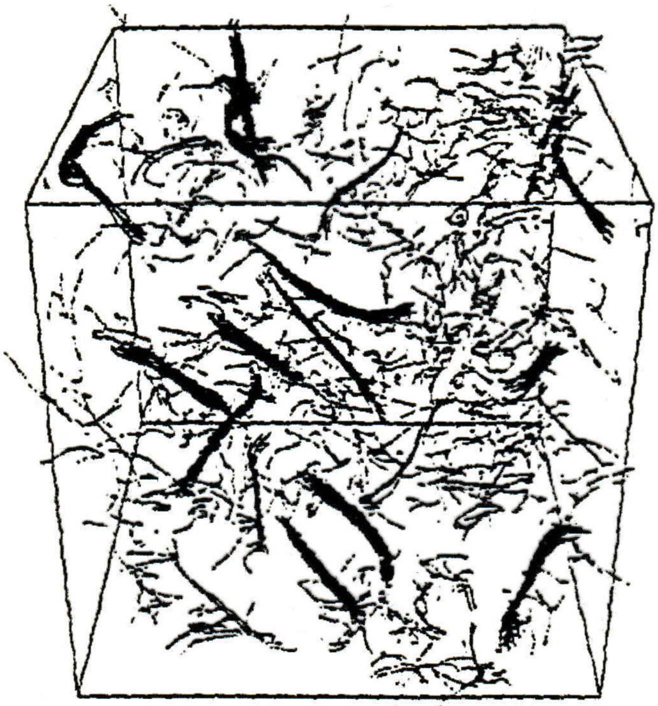
Faible épaisseur : vorticité concentrée
approche analytique

Problème : Evolution ? Interactions?
Stabilité ?

Vorticité gelée

- Utilité de leur étude

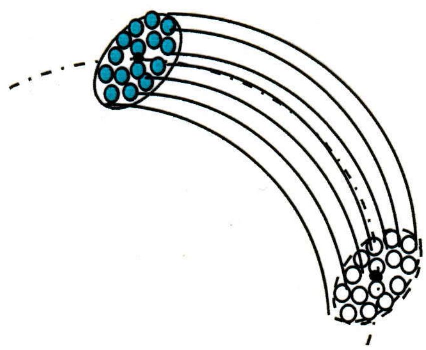
- Simulation Numérique Directe d'une turbulence



Champ de vorticité $Re_\lambda \approx 77$

- Méthodes numériques :

épaisseur non fine



pas de particule de vorticité

• Références bibliographiques : Evolution de filaments fins

- *Etude d'anneaux* **circulaires** :

  Kelvin (1867) , Hicks(1884), Dyson(1893), Lamb(1906), Saffman (1970), Fraenkel (70) Fukumoto (1997)

- *Etude d'anneaux* **non circulaires :**
  - *Méthodes asymptotiques* :

    Levi-Civita (1932), Widnall (1971),

    Moore (1972), **Ting(1978)**, Fukumoto(1991),

    Klein(1991,1995)

  - *Méthodes de 'coupure' : (cut-off)*

    JJ.Thomson 1883, Hama-Burger 62, Crow 70 Levy 28, Rosenhead 30, Parks 70, Saffman 72-92,...

Reste t'il encore des choses à Etudier ?

1) L'équation d'évolution :

Comprendre le DAR

$\Rightarrow$ Calcul formel

Calcul rigoureux des intégrales singulières

2) Oscillations d'un anneau circulaire

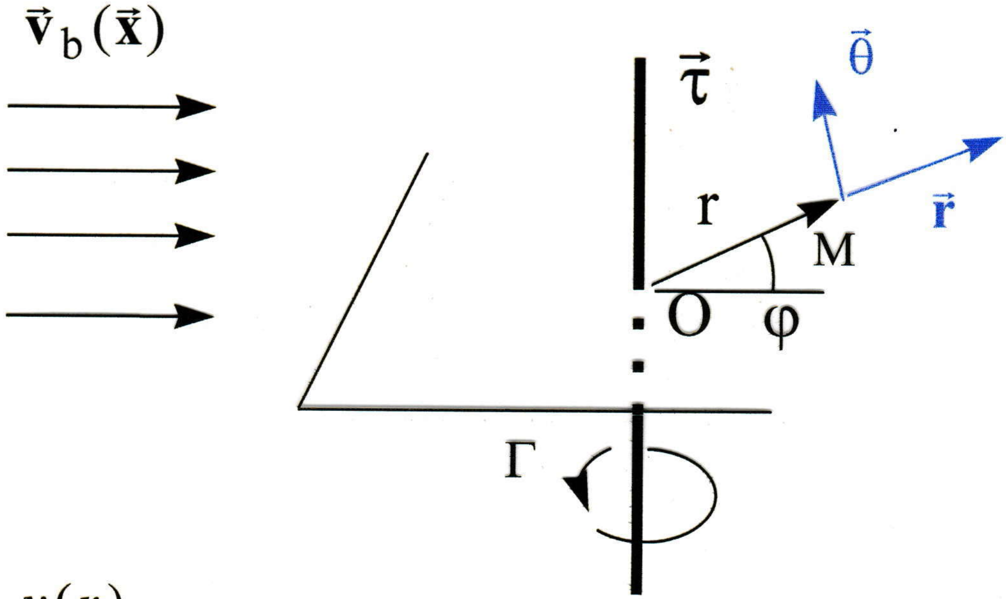
3) Justifier les méthodes de Coupure

4) Oscillations de filaments droits

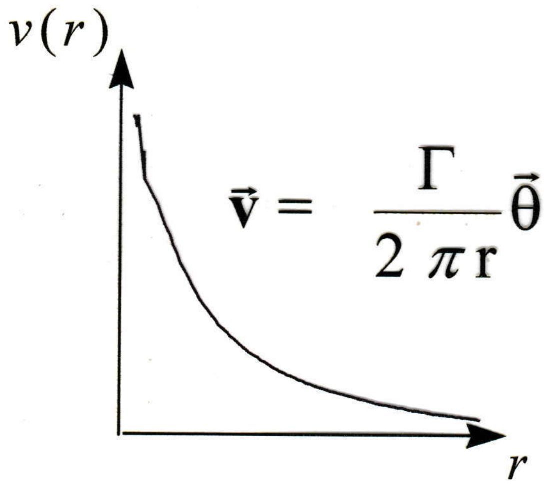
Conclusion et Perspectives

1 Equation d'évolution des filaments tourbillons

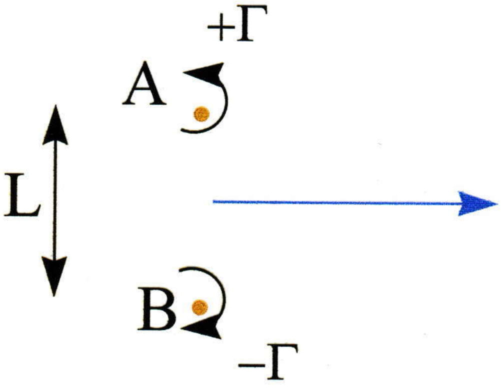
- Le filet tourbillon droit



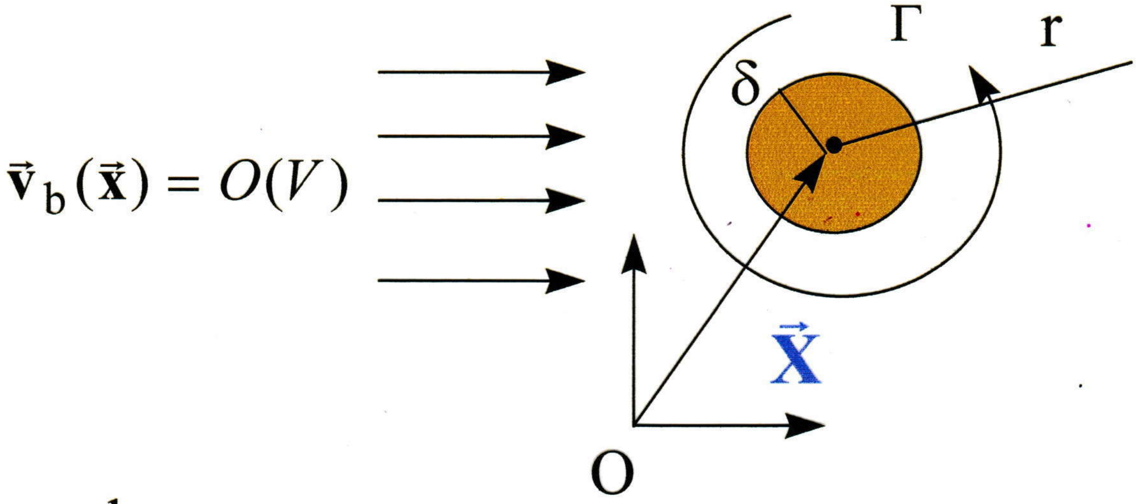
$$\vec{\omega} = \Gamma \delta \vec{\tau}$$



$$\vec{v} = \frac{\Gamma}{2\pi r}\vec{\theta} \qquad \vec{v}(O) = \vec{v}_b(O)$$



$$v(A) = v(B) = \frac{\Gamma}{2\pi L}$$



Deux longueurs :

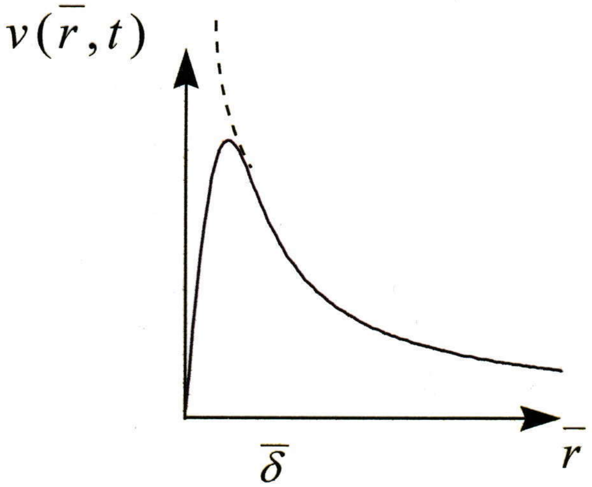
$$\delta = O(\delta_c)$$
$$L = \Gamma / V = O(l_c)$$
avec $\frac{\delta_c}{l_c} \equiv \varepsilon << 1$

$$Re = \frac{\Gamma}{\nu} = \frac{1}{\alpha^2 \varepsilon^2} >> 1 \quad \alpha = O(1) \text{ ou } \alpha = 0$$

variable extérieure : r *variable intérieure :* $\bar{r} = r / \varepsilon$

$$\vec{v} = \dot{\vec{X}} + \vec{V}$$

DAR : couche limite



$$\bar{\delta} = \delta / \varepsilon = \sqrt{4 \Gamma \alpha^2 t}$$

$$\dot{\vec{X}} = \vec{v}_b(\vec{X})$$

Coordinates and geometry

Filament

$$\mathbf{X} = \mathbf{X}(s,t) \qquad T : \text{torsion}$$

$$K : \text{curvature} \qquad (\mathbf{t}, \mathbf{n}, \mathbf{b})$$

Local coordinates

$$\mathrm{M}(r, \varphi, s) \qquad (\mathbf{e}_r, \mathbf{e}_\varphi, \mathbf{t})$$

$$\mathbf{x} = \mathbf{OM} = \mathbf{X}(s,t) + r\mathbf{e}_r(\varphi, s, t)$$

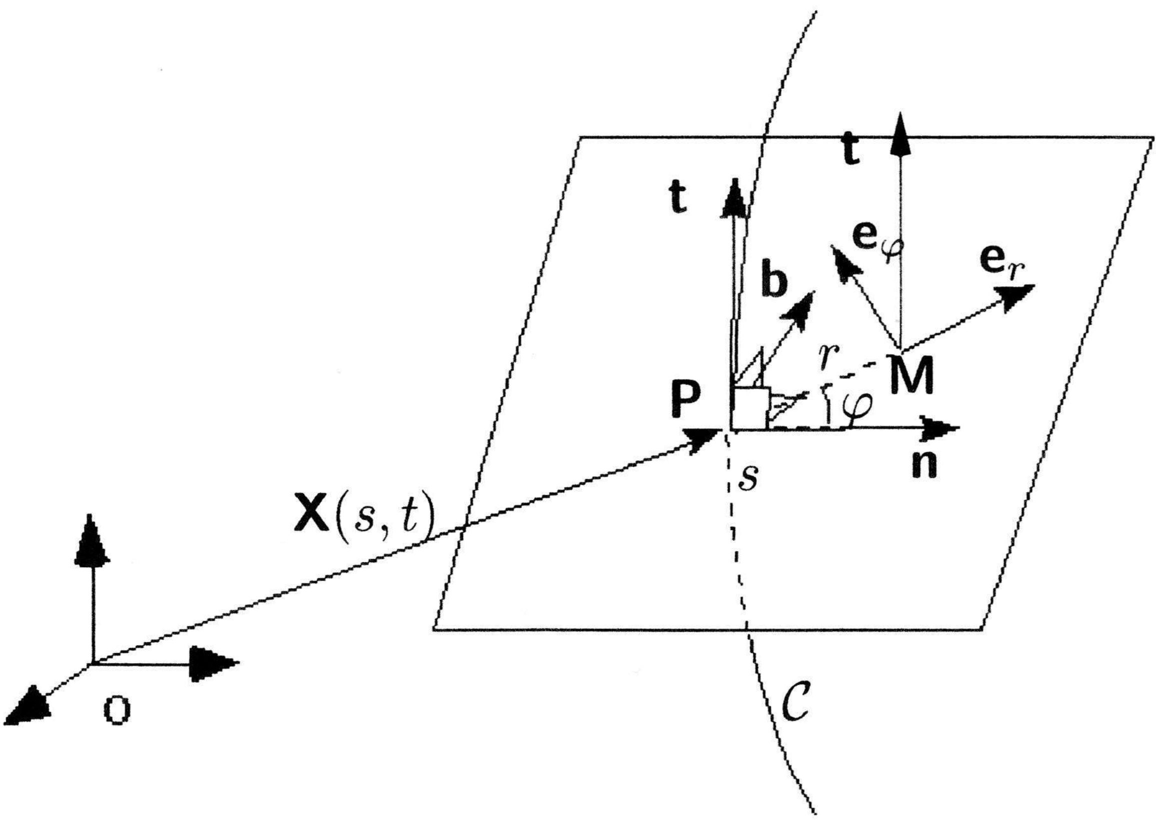


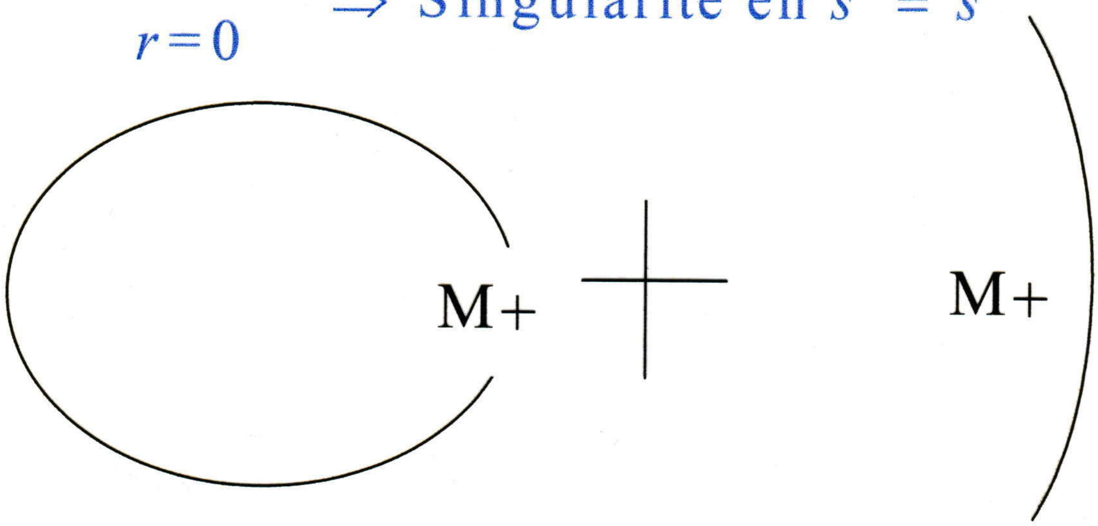
Figure 2: The central curve $\mathcal{C}$ and the local co-ordinates of the scroll filament

• Limite du champ de vitesse sur le filet tourbillon

$$\vec{\mathbf{v}}(\vec{\mathbf{x}}) = \frac{\Gamma}{4\pi} \int_C \frac{\vec{\tau}(s') \wedge (\vec{\mathbf{x}} - \vec{\mathbf{X}}(s'))}{\left|\vec{\mathbf{x}} - \vec{\mathbf{X}}(s')\right|^3} ds'$$

$\vec{\mathbf{x}} = \vec{\mathbf{X}}(s)$ $\Rightarrow$ Singularité en $s' = s$

$r = 0$



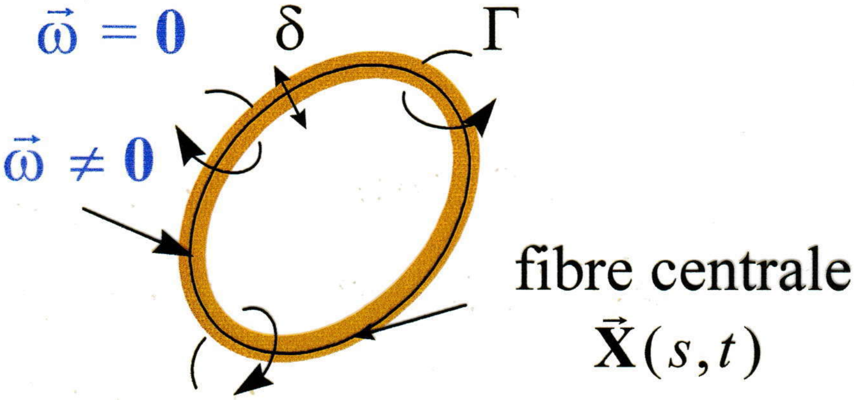
$r \to 0$ à s'-s fixé

$r \to 0$ à $(s'$-$s\,)\,/\,r$ fixé

$$\vec{\mathbf{v}}(r \to 0) = \frac{1}{2\pi r}\vec{\theta} + \frac{K}{4\pi}\left[\ln\frac{S}{r} - 1\right]\vec{\mathbf{b}} + \frac{K}{4\pi}\cos(\varphi)\vec{\theta} + \vec{\mathbf{A}} + O(r\ln r)$$

$$\text{avec } \vec{\mathbf{A}} = \int_C \frac{\vec{\tau}(s') \wedge (\vec{\mathbf{X}}(s) - \vec{\mathbf{X}}(s')}{\left|\vec{\mathbf{X}}(s) - \vec{\mathbf{X}}(s')\right|^3} - \frac{K(s)\vec{\mathbf{b}}(s)}{\left|s' - s\right|} ds'$$

• Les échelles



Deux longueurs :

$$\delta = O(\delta_C) \qquad \text{avec } \frac{\delta_c}{l_c} \equiv \varepsilon << 1$$

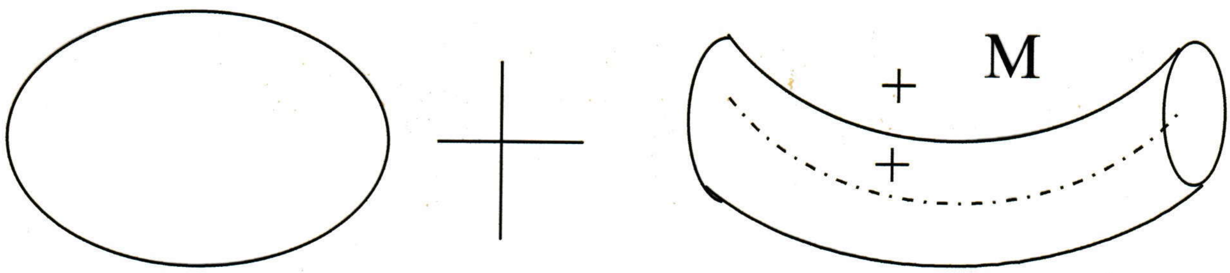
$$R, S, L = O(l_c)$$

$$\mathrm{Re} = \frac{\Gamma}{\nu} = \frac{1}{\alpha^2 \varepsilon^2} >> 1 \qquad \alpha = O(1) \text{ ou } \alpha = 0$$

$$\vec{\mathbf{v}} = \dot{\vec{\mathbf{X}}} + \vec{\mathbf{V}} \qquad \vec{\mathbf{V}} = u\vec{\mathbf{r}} + v\vec{\theta} + w\vec{\tau}$$

$$\dot{\vec{\mathbf{X}}} = ?$$

• Limites extérieure et intérieure



problème extérieur

$\varepsilon \to 0$ à r fixé :

limite extérieure

problème intérieur

$\varepsilon \to 0$ à $\bar{r} = r/\varepsilon$ fixé :

limite intérieure

- Développement extérieur :

$$\vec{\mathbf{v}}^{out} = \vec{\mathbf{v}}^{out^{(0)}}(r,s,t) + \varepsilon \; \vec{\mathbf{v}}^{out^{(1)}}(r,\varphi,s,t) + \ldots$$

- Développement intérieur :

$$\vec{\mathbf{V}}^{inn} = \varepsilon^{-1} \vec{\mathbf{V}}^{inn^{(0)}}(\bar{r},s,t) + \vec{\mathbf{V}}^{inn^{(1)}}(\bar{r},\varphi,s,t) + \ldots$$

- Développement fibre :

$$\vec{\mathbf{X}}(s,t,\varepsilon) = \vec{\mathbf{X}}^{(0)}(s,t) + \varepsilon \; \vec{\mathbf{X}}^{(1)}(s,t) + \ldots$$

$$\dot{\vec{\mathbf{X}}} = f(\vec{\mathbf{X}}, \vec{\mathbf{V}}^{inn}) \qquad \vec{\mathbf{V}}^{inn} = ?$$

- EDP :

$$\vec{\mathbf{V}}^{inn} = \varepsilon^{-1}\vec{\mathbf{V}}^{inn^{(0)}}(\bar{r},s,t) + \vec{\mathbf{V}}^{inn^{(1)}}(\bar{r},\varphi,s,t)+\ldots$$

Les équations de Navier Stokes + Cons. Masse

$$\vec{\mathbf{V}}^{inn^{(i)}}(\bar{r},\varphi,s,t) \qquad \vec{\mathbf{V}} = u\vec{\mathbf{r}} + v\vec{\theta} + w\vec{\tau}$$

Maple

- Conditions aux limites en $\bar{r} = 0$

$$\vec{\mathbf{V}}^{inn^{(i)}}(\bar{r} = 0,\varphi,s,t) = 0$$

- Conditions aux limites en $\bar{r} = \infty$

$$\vec{\mathbf{V}}^{inn^{(i)}}(\bar{r} \to \infty,\varphi,s,t) \quad ?$$

La loi de raccord :

- $$\vec{\mathbf{X}} + \vec{\mathbf{V}}^{\mathrm{inn}}(\bar{r} \to \infty,\varphi,s,t) = \vec{\mathbf{v}}^{\mathrm{out}}(r \to 0,\varphi,s,t)$$

$$\vec{\mathbf{v}}^{out} = \vec{\mathbf{v}}^{out^{(0)}}(r,s,t) + \varepsilon \; \vec{\mathbf{v}}^{out^{(1)}}(r,\varphi,s,t)+\ldots$$

$r \to 0 \qquad r = \varepsilon\,\bar{r}$

• Résolution des équations :

$$\vec{\mathbf{V}}^{inn^{(i)}}(\bar{r},\varphi,s,t) \qquad \vec{\mathbf{V}} = u\vec{\mathbf{r}} + v\vec{\theta} + w\vec{\tau}$$

Séries de Fourier :

$$v^{(i)}(\bar{r},\varphi,s,t) = v_c^{(i)}(\bar{r},s,t) + \sum_n \cos(n\varphi) v_{n1}^{(i)}(\bar{r},s,t) + \sum_n \sin(n\varphi) v_{n2}^{(i)}(\bar{r},s,t)$$

$$\text{Partie}\left|\begin{array}{l}\text{axisymétrique}\\ \text{en } \cos(n\varphi)\\ \text{en } \sin(n\varphi)\end{array}\right| \text{des Equations d'Ordre } i$$

• Partie axisymétrique :

Ordre $i=1$ $\Rightarrow$ Equations de compatibilité

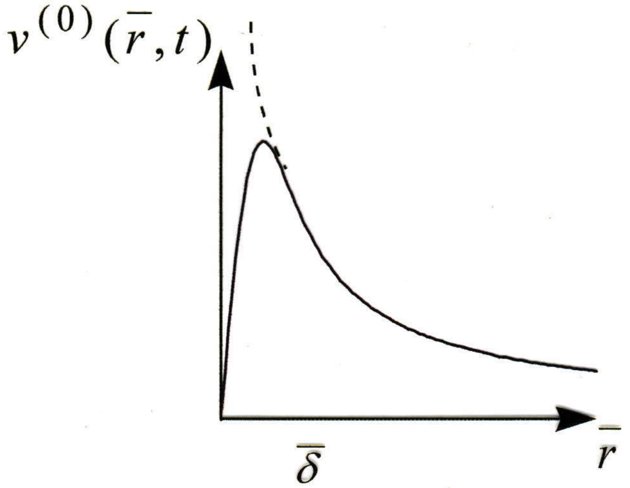
$v_{,s}^{(0)} = w_{,s}^{(0)} = 0 \Rightarrow$ satisfaites

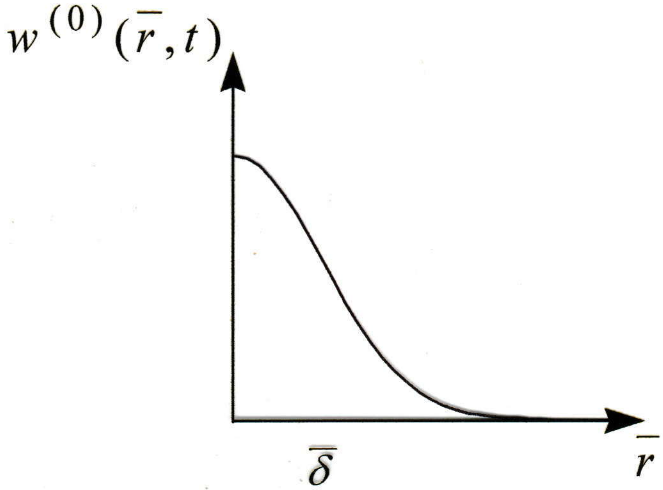
Ordre $i=2$ $\Rightarrow$ $v^{(0)}$ $w^{(0)}$

• Anneau similaire

S(t) est la longueur de l'anneau

$$v^{(0)} = \frac{1}{2\pi\bar{r}}\left(1 - e^{-\left(\frac{\bar{r}}{\bar{\delta}}\right)^2}\right) \qquad w^{(0)} = \frac{m_0}{\pi\bar{\delta}^2}\left(\frac{S_0}{S}\right)^2 e^{-\left(\frac{\bar{r}}{\bar{\delta}}\right)^2}$$

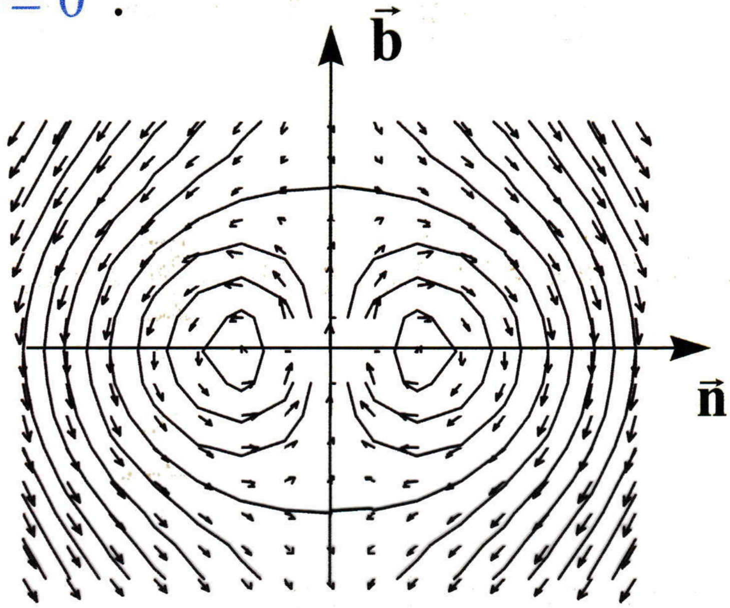




$$\bar{\delta}(t) = \left(\frac{S_0}{S}\right)^{1/2}\left(\bar{\delta}^2(t=0) + 4\alpha^2\int_0^t \frac{S(t')}{S_0}dt'\right)^{1/2}$$

• Partie en $\cos(\varphi)$ et $\sin(\varphi)$ à l'ordre i=1 :

CL en $\bar{r} = 0$:



CL en $\bar{r} = \infty$: $\dot{\vec{\mathbf{X}}}$

Limite $\bar{r} \to \infty$ (Inté.Singu.) | Identification ⇒

$$\dot{\vec{\mathbf{X}}}^{(0)}(s,t) = \vec{\mathbf{Q}}^* + \frac{\Gamma K^{(0)}(s,t)}{4\pi}\left[-\ln\varepsilon + C_v(t) + C_w(t)\right]\vec{\mathbf{b}}^{(0)}$$

$\vec{\mathbf{Q}}^* = \vec{\mathbf{A}} - (\vec{\mathbf{A}} \bullet \vec{\tau})\,\vec{\tau}$

• Partie en $\cos(n\varphi)$ et $\sin(n\varphi)$ à l'ordre i=1 :

CL en $\bar{r} = 0$ CL en $\bar{r} = \infty$

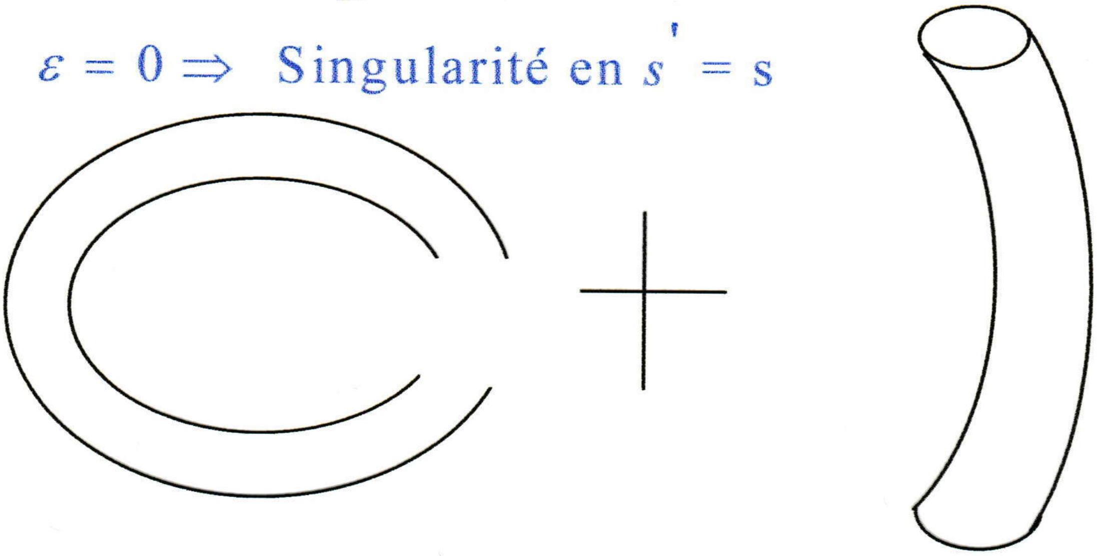
mode n=2

• Limite intérieure dans B&S

$$\vec{\mathbf{v}}(\vec{\mathbf{x}},t,\varepsilon) = \frac{1}{4\pi}\iiint \frac{\vec{\omega}(\vec{\mathbf{x}}',t,\varepsilon)\wedge(\vec{\mathbf{x}}-\vec{\mathbf{x}}')}{\left|\vec{\mathbf{x}}-\vec{\mathbf{x}}'\right|^3}dx'$$

$\varepsilon \to 0$ à $\bar{r} = \frac{r}{\varepsilon}$ fixé

$\varepsilon = 0 \Rightarrow$ Singularité en $s' = s$



$\varepsilon \to 0$ à s'-s fixé $\qquad$ $\varepsilon \to 0$ à (s'-s) / ε fixé

$$\vec{\mathbf{v}}^{inn}(\bar{r},\varphi,s,\varepsilon) = \frac{1}{\varepsilon}\iint_{\text{Section}} \mathrm{F}\left(\vec{\omega}^{(0)}(\bar{r}',\varphi',s)\right)\bar{r}'d\bar{r}'d\varphi' \quad \text{B\&S 2D}$$

$$+ \frac{K}{4\pi}\left[\ln\frac{S}{\varepsilon}-1\right]\vec{\mathbf{b}} + \vec{\mathbf{A}}(s) + \iint_{\text{Section}} \tilde{\mathrm{F}}\left(\vec{\omega}^{(0)}(\bar{r}',\varphi',s)\right)\bar{r}'d\bar{r}'d\varphi'$$

$$+ O(\varepsilon\ln\varepsilon)$$

$\bar{\mathrm{r}} \to \infty \quad \Rightarrow \quad \vec{\mathbf{v}}^{inn}(\bar{r}\to\infty,\varphi,s,\varepsilon)$ CL en ∞

- Recherche d'un ordre Supérieur :

Maple But : cohérence

-Partie axisymétrique :

Ordre i=2 $\Rightarrow$ $v^{(0)}$ $w^{(0)}$

$\Rightarrow$ Equations de compatibilité

Hypothèses : $v_c^{(1)}$ indépendant de s

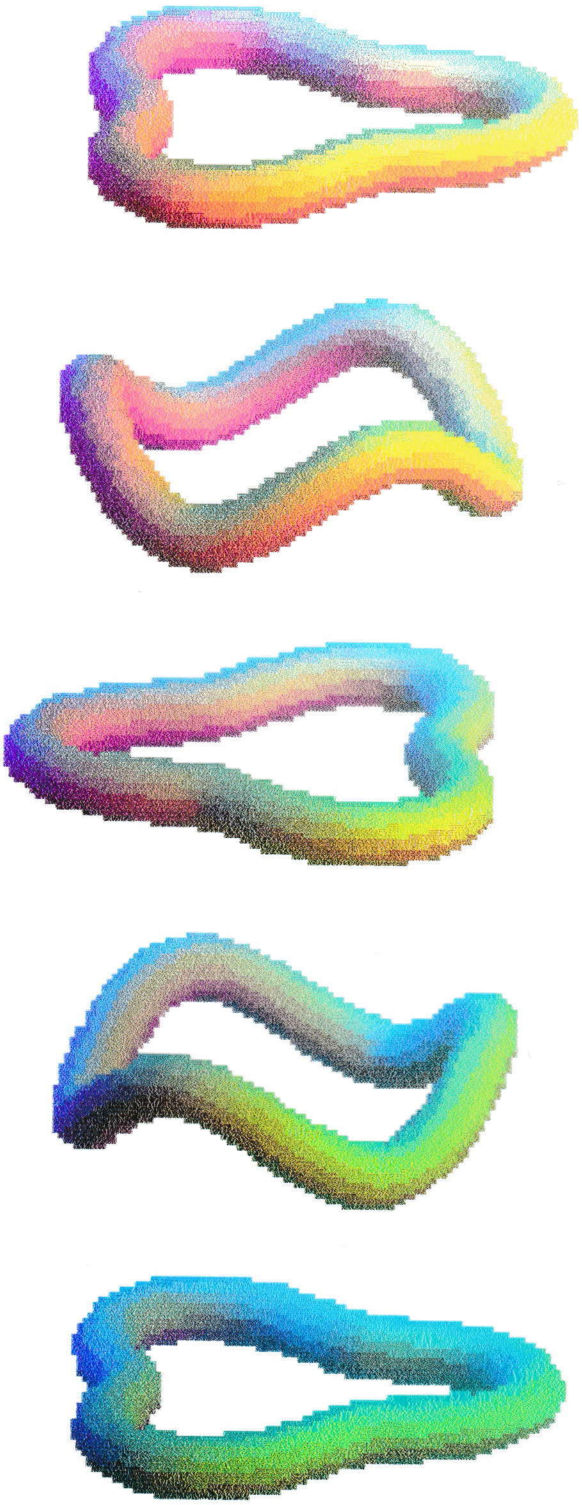
$$\frac{\partial w_c^{(1)}(\bar{r},s,t)}{\partial s} = -\overset{\circ}{\sigma}^{(0)} + a(s)\sigma^{(0)}$$

avec $a(s) = \overset{\circ}{S}^{(0)} / S^{(0)}$

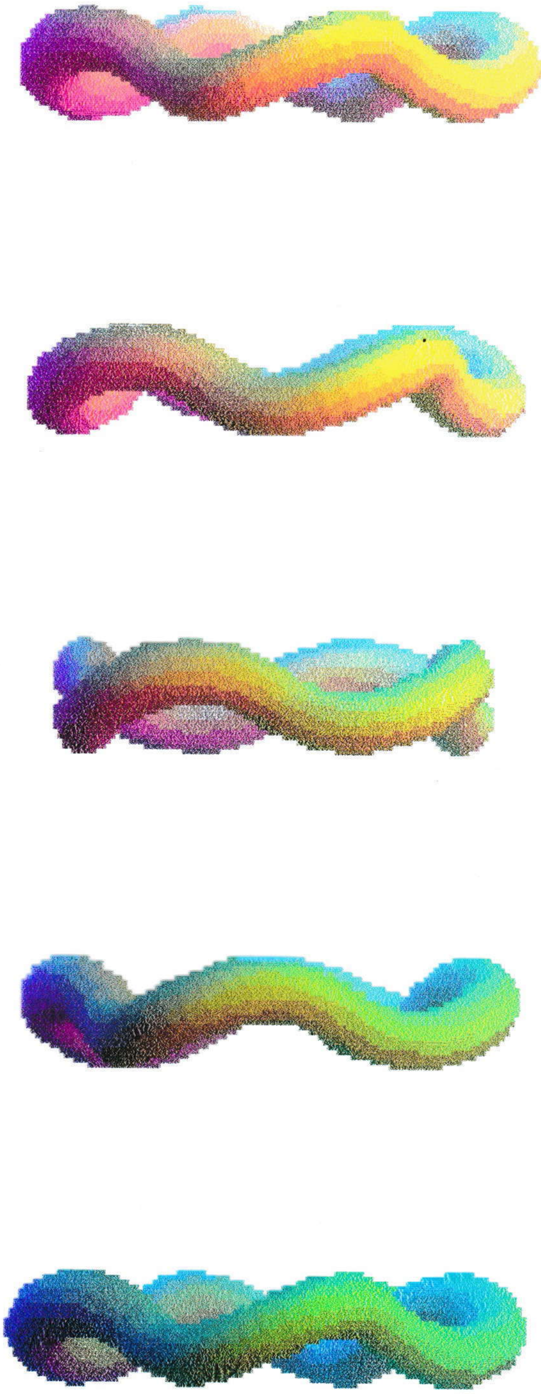
Ordre i=3 $\Rightarrow$ $v_c^{(1)}$ $w_c^{(1)}$

- Partie en $\cos(\varphi)$ et $\sin(\varphi)$ à l'ordre i=2 :

$\Rightarrow$ $\dot{\vec{\mathbf{X}}}^{(1)} = f(\vec{\mathbf{X}}^{(1)}, \vec{\mathbf{V}}^{inn})$

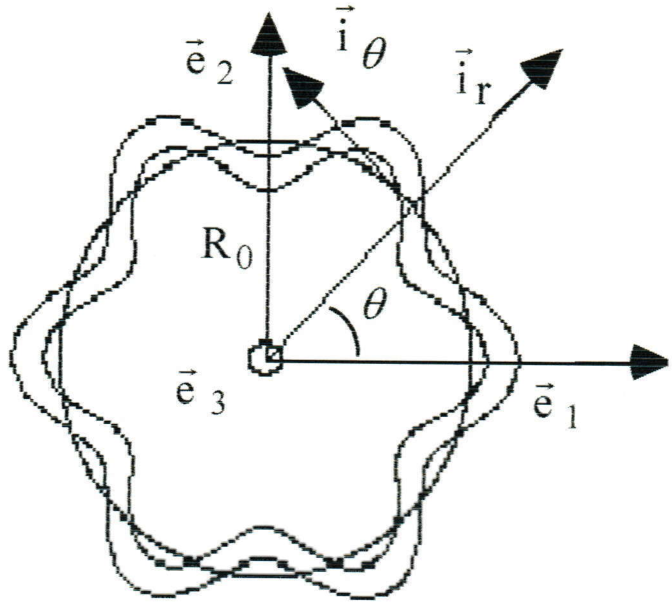


mode $n = 3$



mode $n = 4$

2 Oscillations d'un anneau circulaire perturbé



mode m

- Etude linéaire d'un anneau circulaire :

  Calcul de la Stabilité de l'anneau :

  JJ.Thomson 1882- Widnall 73

  Notre travail :

  - Complément de l'étude de Widnall : partie stable oscillante
  - Cas Visqueux et Vitesse axiale

- Période d'oscillation du mode n :

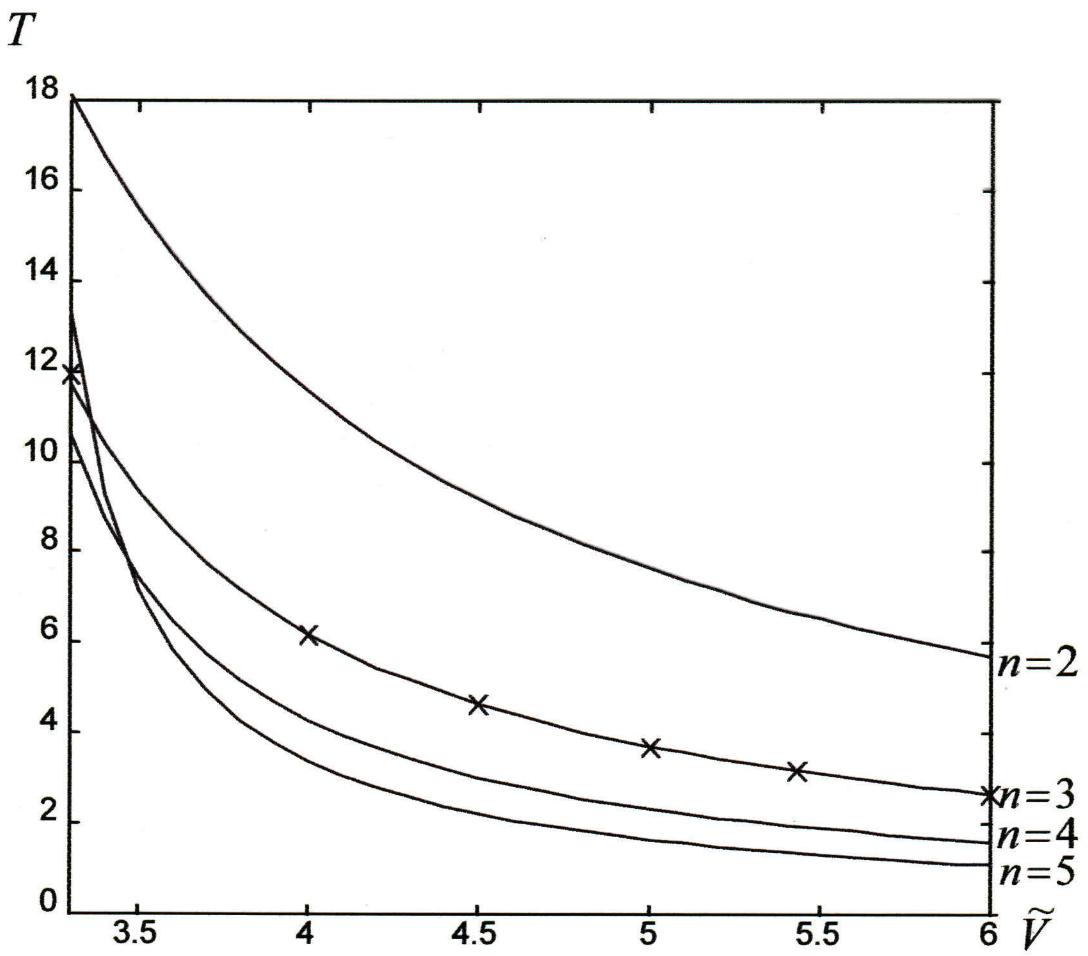
$$T = 8\pi^2 R_0^2 \Big/ \sqrt{\left[n^2\widetilde{V} - g_\xi(n)\right]\left[(n^2-1)\widetilde{V} + g_\rho(n)\right]}$$

$$\widetilde{V} = \ln(8/\varepsilon) - 1/2 + A - 2m_0^2 \qquad A = -0.058$$

n	2	3	4
$g_\xi(n)$	6.66	20.8	43.123
$g_\rho(n)$	-7	- 21.33	- 43.8

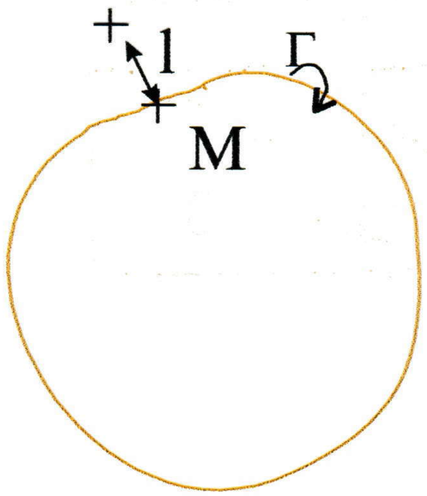
n=2 DSB

- Comparaison :



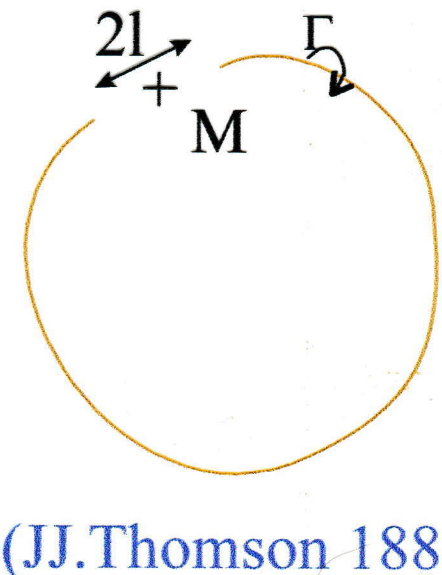
3
Les méthodes de Coupure et leur justification

- JJ.Thomson 1883 :



$$\dot{\vec{\mathbf{X}}} = \vec{\mathbf{v}}_{\mathrm{Dirac}}(l)$$

- Burger :



$$\dot{\vec{\mathbf{X}}} = \frac{\Gamma}{4\pi} \int_{l}^{S-l} \frac{\vec{\tau}' \wedge (\vec{\mathbf{x}} - \vec{\mathbf{x}}')}{\left|\vec{\mathbf{x}} - \vec{\mathbf{x}}'\right|^3} ds'$$

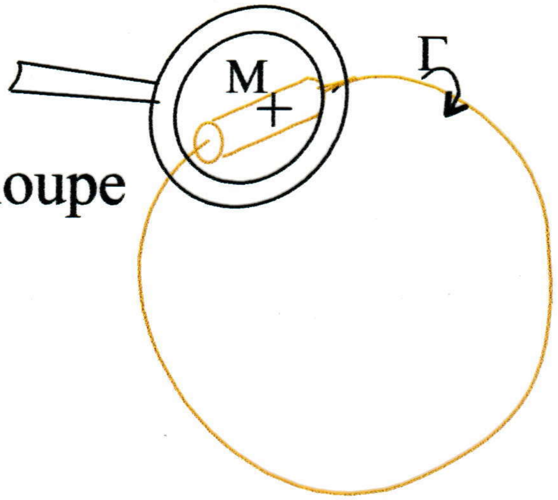
(JJ.Thomson 1883,Hama-Burger 62,Crow 70)

– Rosenhead :

$$\dot{\vec{\mathbf{X}}} = \frac{\Gamma}{4\pi}\int_0^S \frac{\vec{\tau}' \wedge (\vec{\mathbf{x}} - \vec{\mathbf{x}}')}{((\vec{\mathbf{x}} - \vec{\mathbf{x}}')^2 + l^2)^{3/2}} ds'$$

(Levy 28,Rosenhead 30,Parks 70,Saffman 72-92)

– Autres :



$$\dot{\vec{\mathbf{X}}} = \frac{\Gamma}{4\pi}\int_0^S \frac{\vec{\tau}' \wedge (\vec{\mathbf{x}} - \vec{\mathbf{x}}')}{\left|\vec{\mathbf{x}} - \vec{\mathbf{x}}'\right|^3} f\left(\frac{\left|\vec{\mathbf{x}} - \vec{\mathbf{x}}'\right|}{l}\right) ds'$$

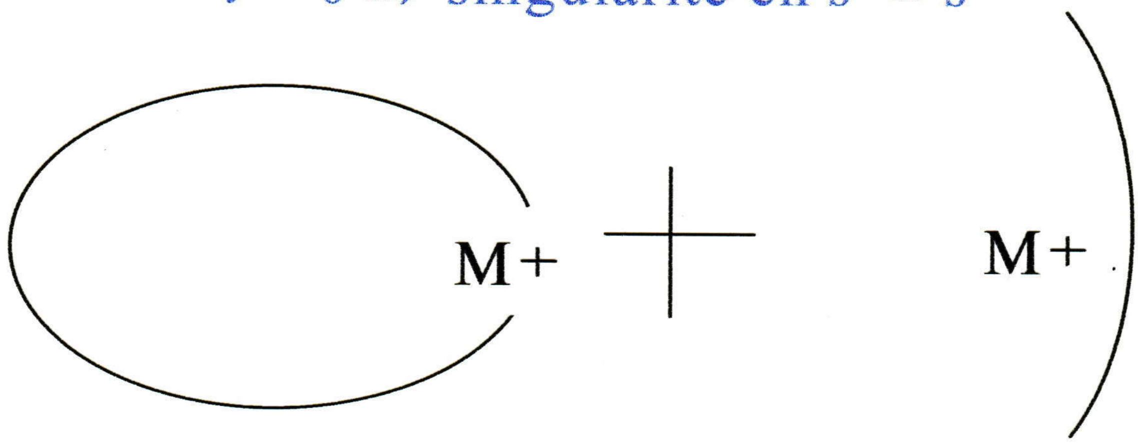
avec $f(r) \longrightarrow 1$ quand $r \longrightarrow \infty$

CHOIX l=ε

Bon choix ?

•Justification des méthodes de Coupure

$l = 0 \Rightarrow$ singularité en $s' = s$



l=0 à s' fixé

l=0 à (s'-s)/ l fixé

Identification : Cut-off ↔ Equation Asymp.

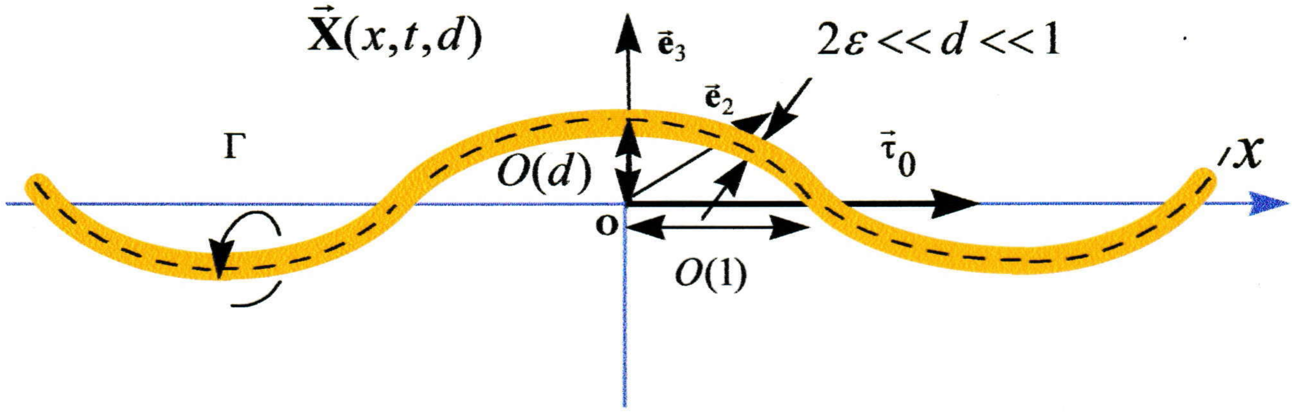
$$l = \varepsilon \quad \exp\left(N - C_v(t) - C_w(t)\right)$$

Nom de la méthode de cut-off	N
Burger	1-ln2
Thomson	1/2
Rosenhead	0
VEM1	0
VEM2	1-0.009122-ln2

$$l = s_c(s)\sigma(\mathrm{s})$$

4 Le filament droit

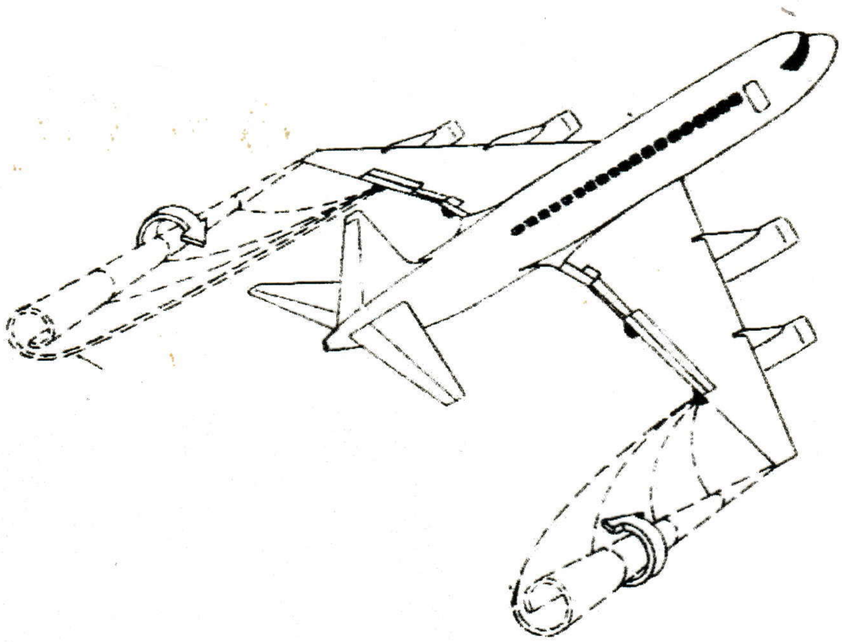
- Période d'oscillation d'un filament droit :

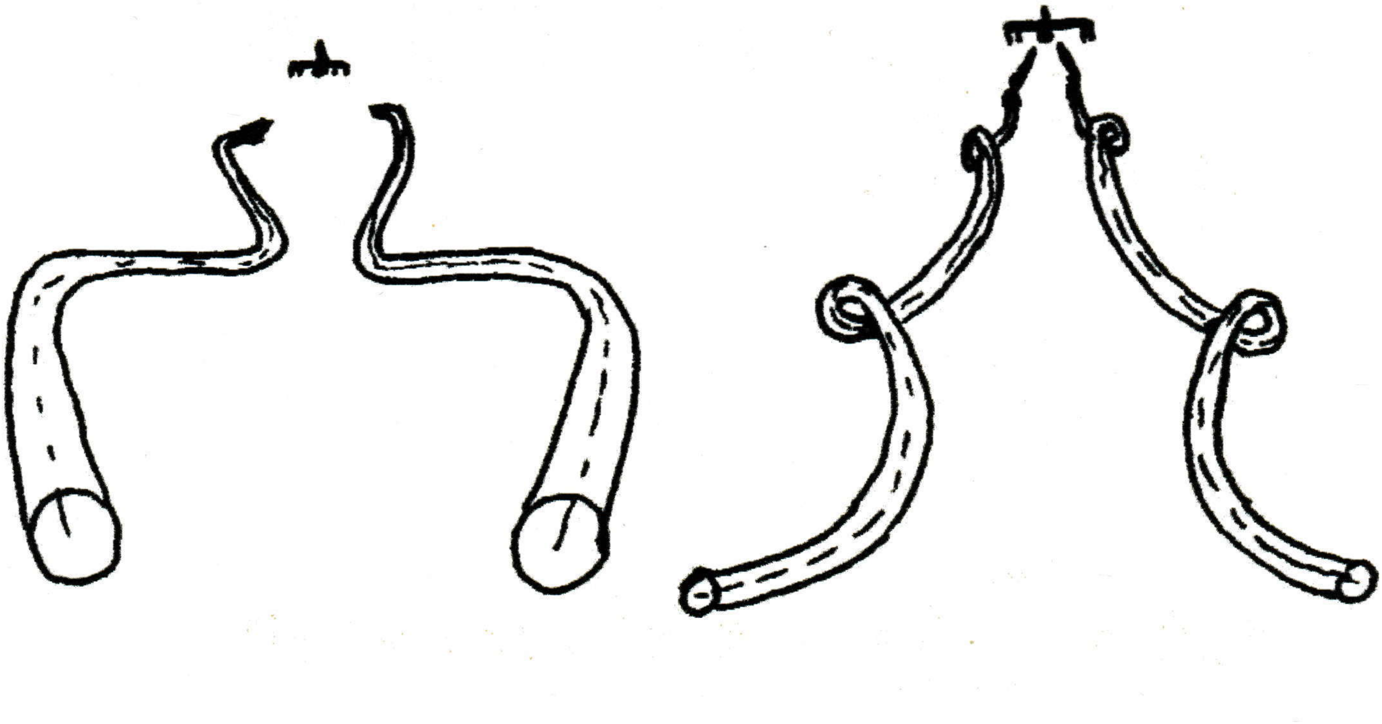


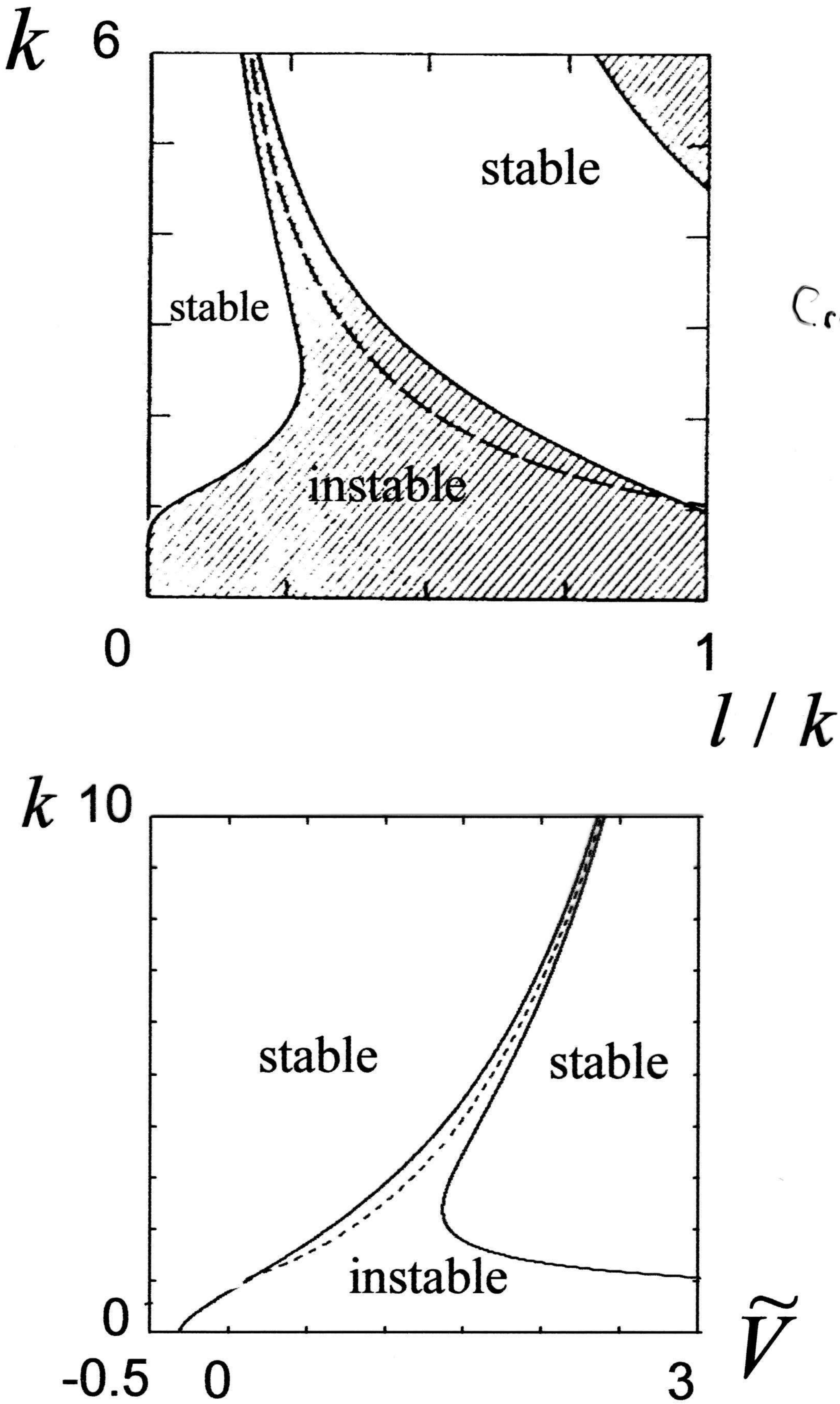
$$T = \frac{8\pi^2}{\left|k^2(\tilde{V} + \frac{1}{2} - \gamma - \ln k)\right|}$$

$$\tilde{V} = A - 1/2 + \ln 2/\varepsilon - 2m_0^2 \qquad A = -0.058$$

- Stabilité de deux filaments parallèles :






k
6
stable
stable
instable
0
1
l / k
k
10
stable
stable
instable
0
-0.5
0
3
$\widetilde{V}$

Crou (70)

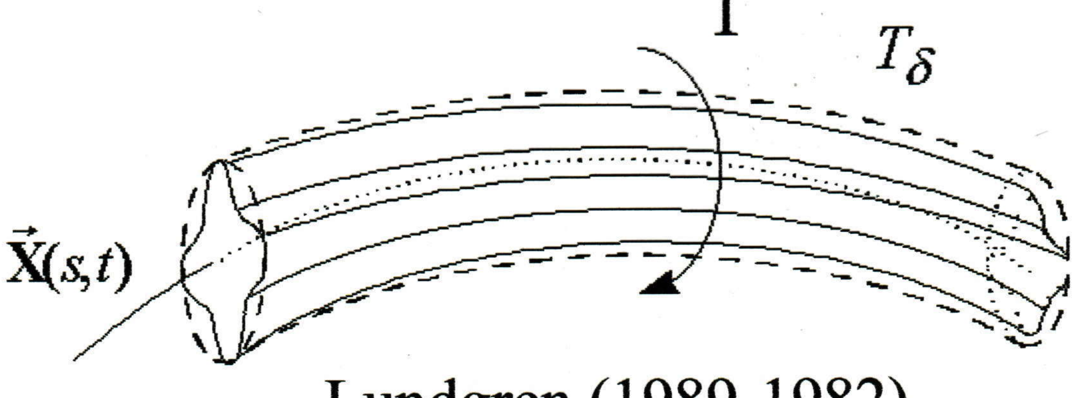
Perspectives

- Nous avons aussi traité :
  étirement , paramétrage fibre centrale,
  écoulement potentiel, termes ln, Klein et Majda
  φ et pas θ

- Ordres supérieurs
  anneau circulaire   Fukumoto (1997)

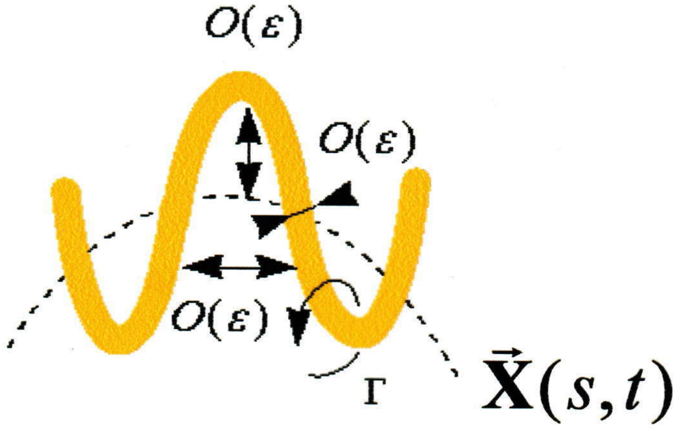
- L'Helicité et son évolution :
  Moffatt (1969), Ricca (1992)

- Variation axiale ou Non Axisymétrique :



Lundgren (1989-1982)

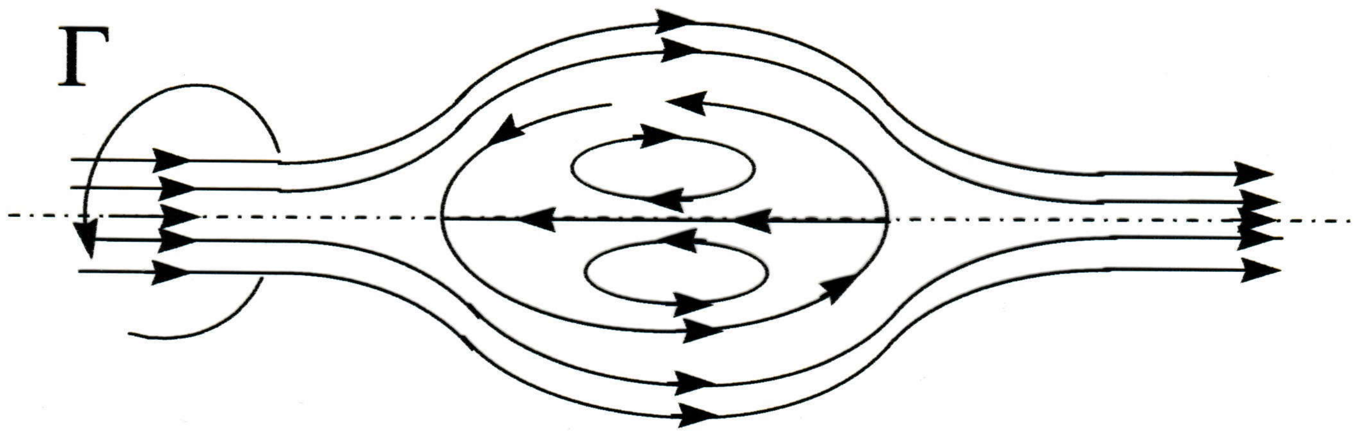
- CLI et Ondes courtes :



$$\bar{s} = \frac{s}{\varepsilon}$$

Tsai et Widnall (1977)

- Eclatement Tourbillonnaire :



Leibovich(1986), Sychev(1993),Schmitz(1996)

- Turbulence ....

Code Maple : déjà écrit