Application of a slender vortex filament code to the study of a four-vortex wake model

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C-Wake european project GRD1-1999-10332

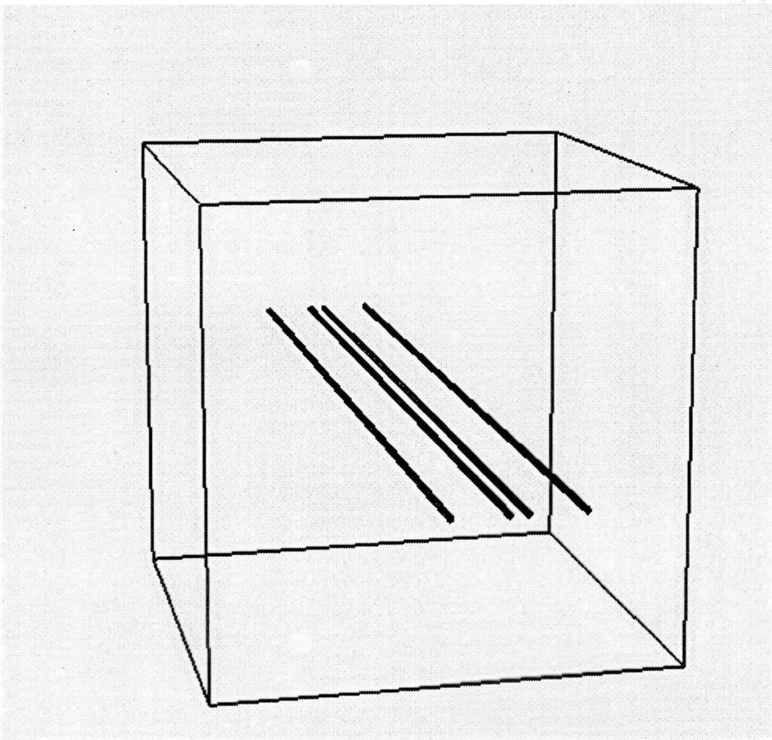
Papers

1. Implementation and validation of a slender vortex filament code, *submitted to Int. J. Numer. Meth. Fluids.*

2. EZ-Vortex documentation: a slender vortex filament solver.

3. Effects of simple generic configuration on near to far field wake by using 3D vortex filament method, C-Wake *partner report UPS-PR 2.2.3-2.*

Introduction:

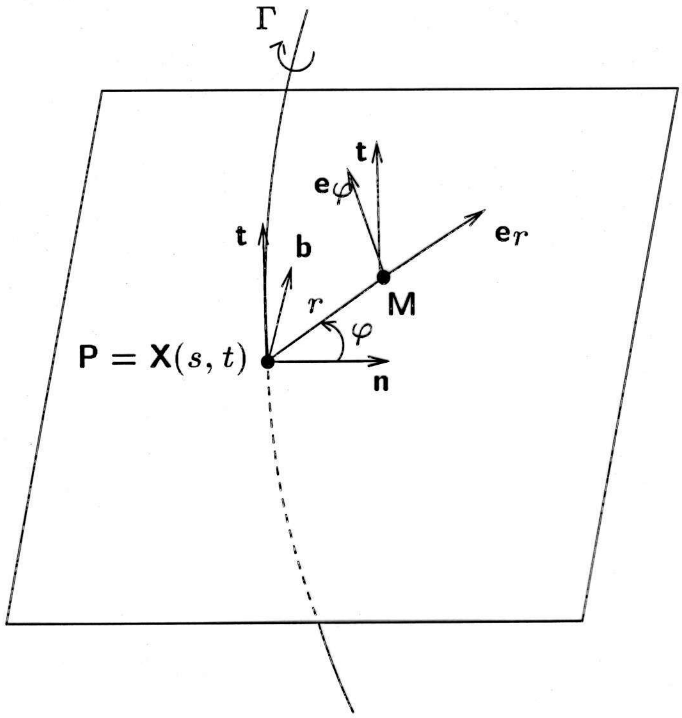


The four-vortex wake

- Rennich and Lele (AIAA 1998): simple vortex filament method
- Fabre and Jacquin (Phy. Fluids 2000): linear stability study
- Klein and Knio (J. Comput. 2000): new improved numerical schemes for slender vortex simulations (vortex ring)

$\Longrightarrow$ implement the Klein and Knio scheme for open filaments (periodicity)
$\Longrightarrow$ comparison with linear stability results

Equation of motion of slender filaments



The vortex filament.

Cut-off Equations

$$\partial \mathbf{X}/\partial t = \frac{1}{4\pi}\int_I \sigma(s',t)\frac{\mathbf{t}(s',t)\times(\mathbf{X}(s,t)-\mathbf{X}(s',t))}{|\mathbf{X}(s,t)-\mathbf{X}(s',t)|^3}ds'$$

where $I = [0, 2\pi[\setminus[s - s_c, s + s_c[$ $\quad s_c$: cut-off length

$\Longrightarrow$ Ad-hoc method. How to chose s_c ?

$\Longrightarrow$ Callegari and Ting: Siam Applied Math. 78

$$s_c(s,t) = \epsilon \exp\left(1 - \ln 2 - C_v(t) - C_w(t)\right)/\sigma(s,t)$$

The equation of Callegari and Ting

$$\boxed{\partial \mathbf{X}/\partial t = \mathbf{A} + \frac{\Gamma K(s,t)}{4\pi} B\mathbf{b}(s,t),}$$

where

$$
\begin{aligned}
\mathbf{A}(s,t) &= \frac{\Gamma}{4\pi}\int_{-\pi}^{+\pi} \sigma(s+s',t)\mathbf{N}\mathrm{d}s', \\
\mathbf{N} &= \frac{\mathbf{t}(s+s',t)\times(\mathbf{X}(s,t)-\mathbf{X}(s+s',t))}{|\mathbf{X}(s,t)-\mathbf{X}(s+s',t)|^3} \\
&\quad -\frac{K(s,t)\mathbf{b}(s,t)}{2\,|\lambda(s,s',t)|}, \\
\sigma(s,t) &= |\partial \mathbf{X}/\partial s|, \\
\lambda(s,s',t) &= \int_{s}^{s+s'} \sigma(s^*,t)\mathrm{d}s^*,
\end{aligned}
$$

and

$$B = -\log\epsilon + \log(S) - 1 + C_v(t) + C_w(t)$$

K: local curvature

S: length of the filament

ϵ: small dimensionless thickness

Similar vortex core

$$C_v(t) = [1+\gamma-\ln 2]/2 - \ln(\bar{\delta}),$$
$$C_w(t) = -2(S_0/S)^4(m_0/(\Gamma\bar{\delta}))^2,$$

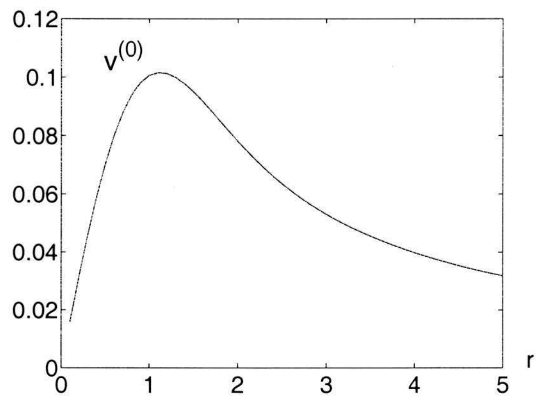
where

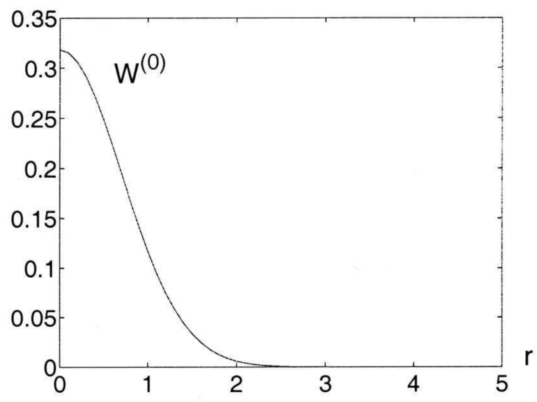
$$\bar{\delta}^2(t) = \bar{\delta}_0^2\left(\frac{S_0}{S(t)}\right)\left(1+\frac{\bar{\delta}_{\bar{\nu}}^2}{\bar{\delta}_0^2}\right)$$
$$\bar{\delta}_{\bar{\nu}}^2 = 4\bar{\nu}\int_0^t \frac{S(t')}{S_0}\mathrm{d}t',$$

$\gamma=$ Euler number. Subscript 0 stands for initial.
$\bar{\delta}=\delta/\epsilon$: the stretched radius
$\bar{\nu}=\nu/\epsilon^2$ is the stretched kinematic viscosity.
m_0: the initial axial flux of the ring.

Circumferential $v^{(0)}$ and axial $w^{(0)}$ velocities:

$$v^{(0)} = \frac{\Gamma}{2\pi\bar{r}}\left[1-e^{-(\bar{r}/\bar{\delta})^2}\right], \quad w^{(0)} = \frac{m_0}{\pi\bar{\delta}^2}\left(\frac{S_0}{S}\right)^2 e^{-(\bar{r}/\bar{\delta})^2},$$

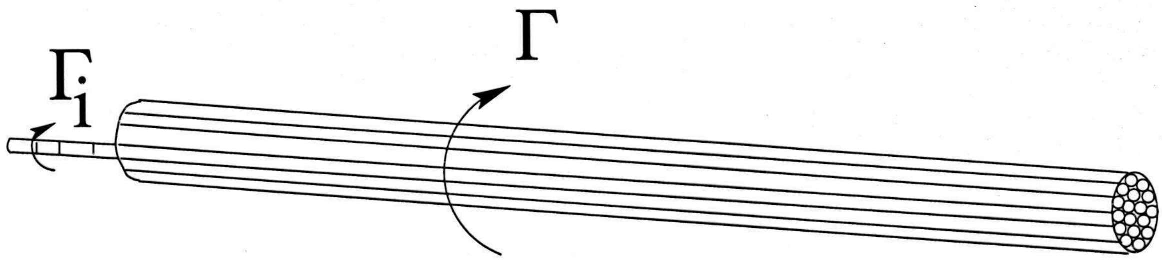




$\bar{r}=r/\epsilon$: the stretched radial distance to the filament,

Vortex Methods / Slender vortex Methods

1) Vortex Blob or Vortex filament Methods



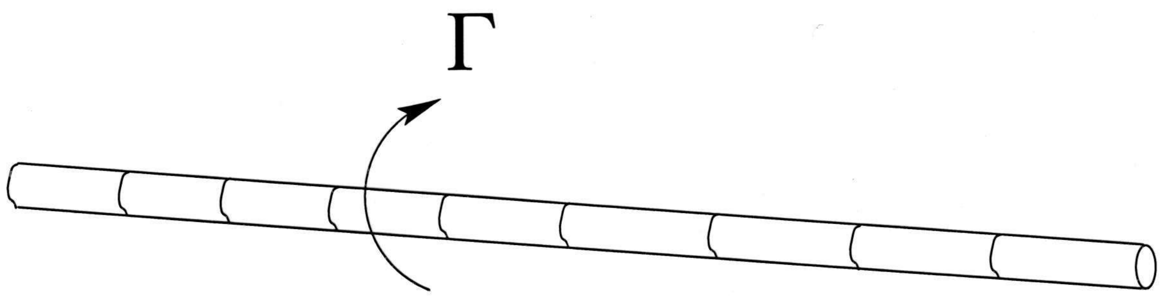
The computation with 1 filament per section is not correct (Klein and Knio JFM 95)

$\Longrightarrow$ Great number of filaments per section to converge

or

$\Longrightarrow$ Theorical correction of the method (Klein and Knio JFM 95/Klein and Knio 2000): new slender vortex solver

2) Slender Vortex filament solver:



+ Nbr filaments per section =1
+ Not stiff with thickness:

boundary layer solved theoretically

$\Longrightarrow$ Small thickness, no reconnection, no short wavelengths

M1 method of Knio and Klein

$$\partial \mathbf{X}/\partial t = \mathbf{v}_{\sigma_1} + (\mathbf{v}_{\sigma_1} - \mathbf{v}_{\sigma_2}) \frac{\log(\sigma_1/\delta^{ttm})}{\log(\sigma_2/\sigma_1)}$$

where

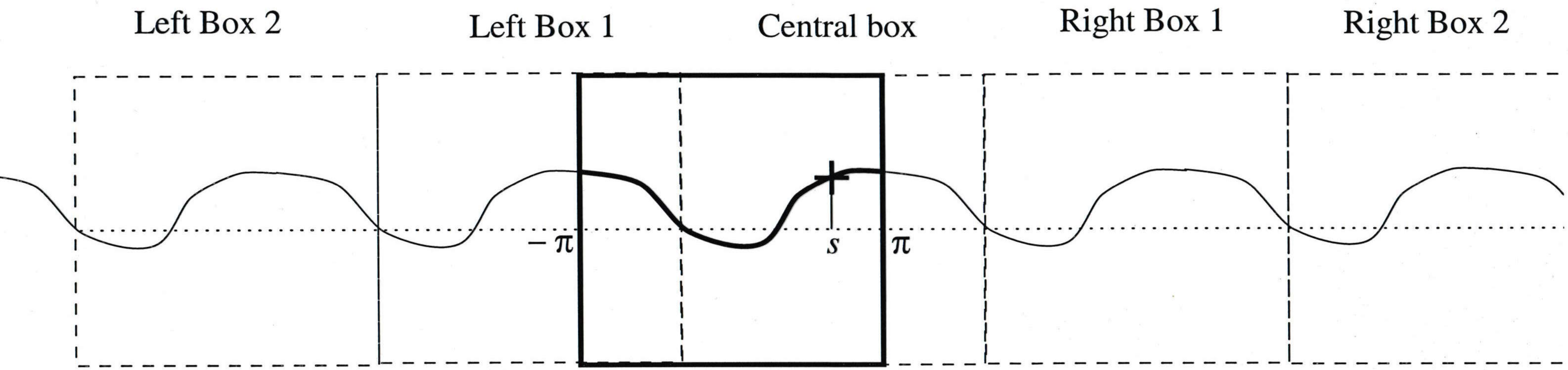
$$\begin{aligned}
\mathbf{v}_x &= \frac{\Gamma}{4\pi} \int_{\mathcal{C}} \sigma(s',t) \mathbf{N}_x \mathrm{d}s', \\
\mathbf{N}_x &= \frac{\mathbf{t}(s',t) \times (\mathbf{X}(s,t) - \mathbf{X}(s',t))}{\left[|\mathbf{X}(s,t) - \mathbf{X}(s',t)|^2 \right]^{3/2}} \kappa\left(\frac{|\mathbf{X}(s,t) - \mathbf{X}(s',t)|}{x}\right) \\
\kappa(r) &= \tanh(r^3)
\end{aligned}$$

and

$$\begin{aligned}
\sigma_1 &= 3\sigma_{max}, \\
\sigma_2 &= 2\sigma_{max}, \\
\sigma_{max} &= \mathrm{d}s \max_{s \in [0,2\pi]} \sigma(s,t). \\
\delta^{ttm} &= \epsilon \exp\left(C^{ttm} + 1 - C_v(t) - C_w(t) \right),
\end{aligned}$$

With the choice of $\kappa(r) = \tanh(r^3)$, the C^{ttm} constant is $C^{ttm} = -0.4202$.

Implementation: open filament (periodic)



Periodic part

code EZ-vortex : - M1 méthod / CST equation / LIA

↳ fast, simple, well documented, easy to handle.

- Open GL
- ≠ vorticity profiles, viscosity: ν
  axial flux

EZ-vortex[1]: a Slender Vortex Filament solver (SVF)

1) Input: Initial position of the filaments

2) Solver: SVF solver for *closed* or *open* filaments

- Equation:
  - The Callegari and Ting Equation (Implicit stepping)
  - The M1 method of Knio and Klein (Adams-Bashforth or Implicit)
- Spatial derivatives: Finit. Diff. or Spectral
- Core: similar core or non-similar (Laguerre series)
- Inviscid or viscous

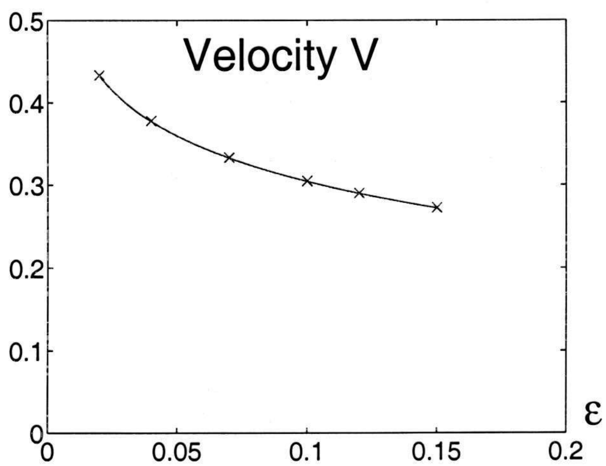
3) Output:

- Run-time drawing with OpenGL (SGI or linux)
- File *history.dat* and mode 'Visualisation' of this file
- Movie of the simulation

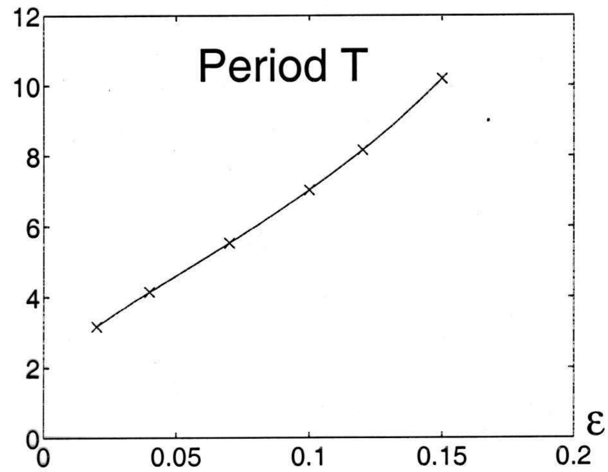
[1] D.Margerit, A.Giovannini, P. Brancher: EZ-vortex documentation: a Slender Vortex Filament solver

Validation

Linear stability analysis of a preturbed circular vortex ring (similar vortex and inviscid):

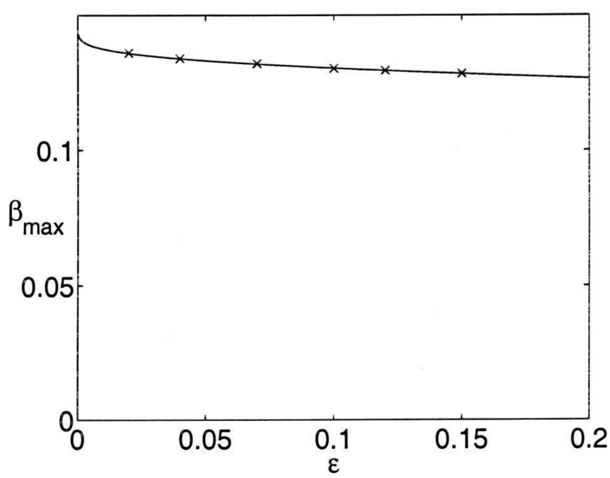


Velocity $V = f(\epsilon)$

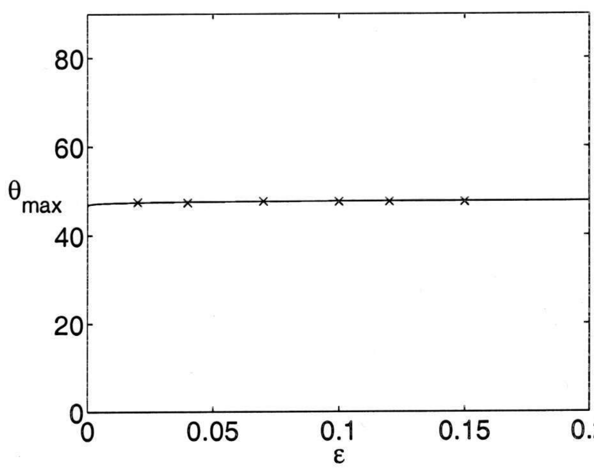


Period $T = f(\epsilon)$ for mode 3

Linear stability analysis of a pair of contra-rotating vortex filaments (similar vortex and inviscid):



Ampl. rate $\beta_{max} = f(\epsilon)$



Planar angle $\theta_{max} = f(\epsilon)$

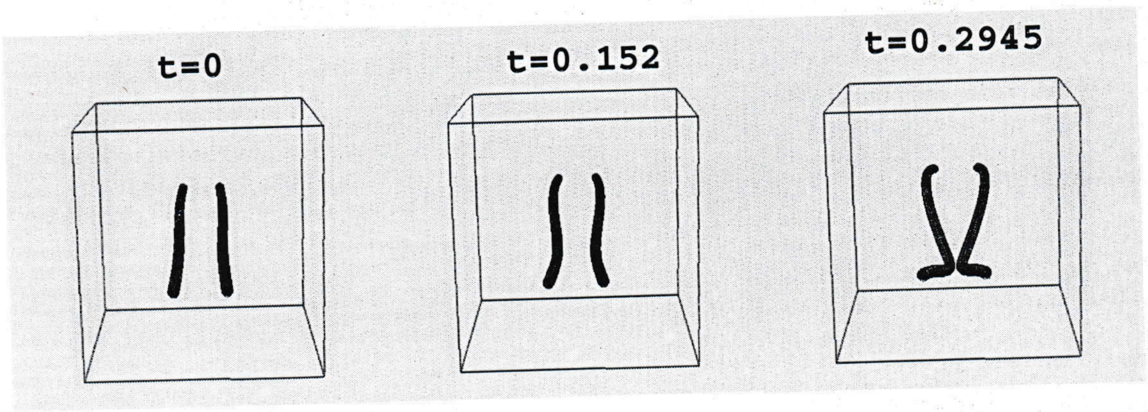
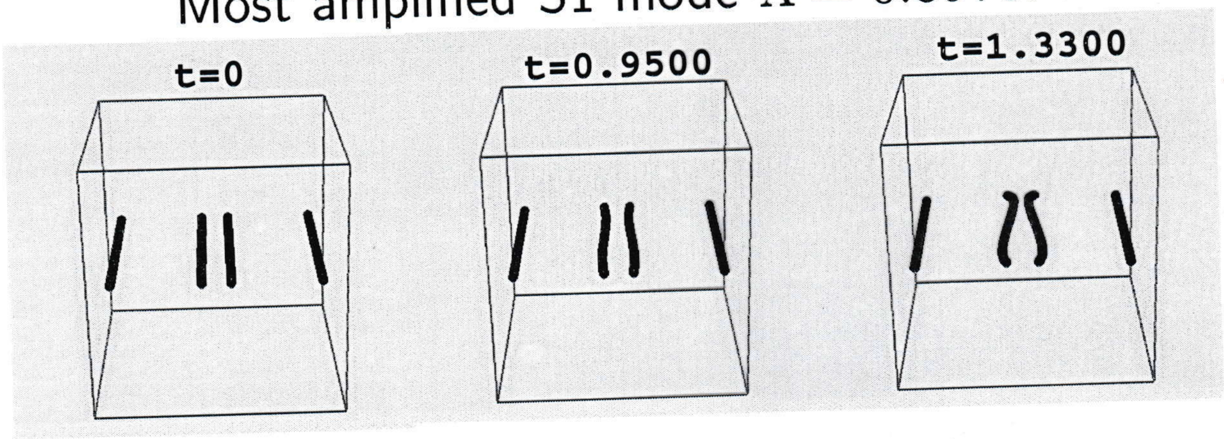


Figure 1: $\Lambda = 10.21$, initial amplitude $\rho = 0.05$, initial thickness $\epsilon = 0.02$ and initial angle $\theta(t = 0) = 47.63(\text{deg})$.

Most amplified S1 mode $\Lambda = 0.8976$.



Long-wave S1 mode $\Lambda = 7.85$.

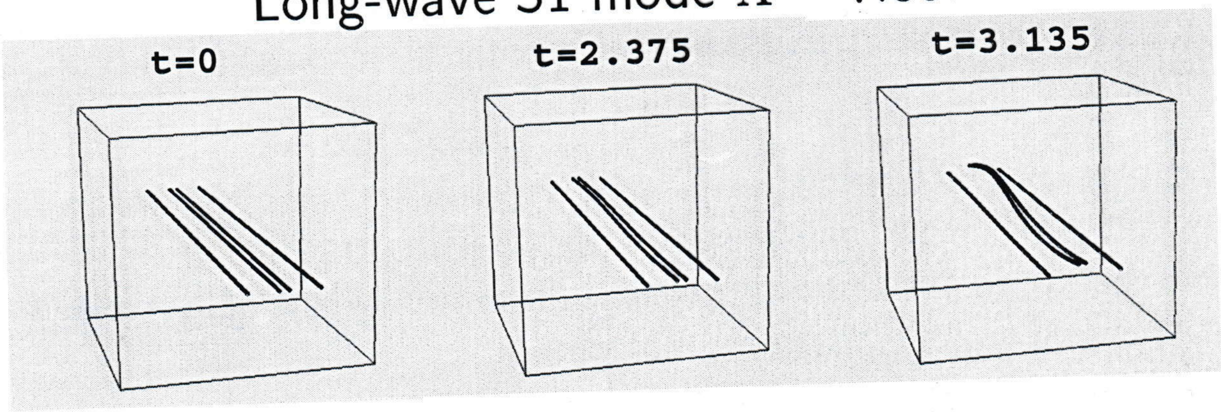


Figure 2: Initial amplitude $\rho_0 = 0.001$ and initial thickness $\epsilon = 0.1$.

Table 1: Four-vortex modes: linear stability (th.) and EZ-vortex (num.) results at $\epsilon = 0.1$.

	Λ	β	θ_o(deg)	θ_i(deg)	$\varrho = \rho_i/\rho_o$	ρ_o/ρ_i %
Most amplified S1 mode (th.)*	0.8976	2.91	105.86	131.24	57.4	1.7
// (num.)	//	2.94	111.04	130.20	52.8	1.9
Long-wave S1 mode (th.)*	7.85	1.55	145.45	103.85	9.72	10.3
// (num.)	//	1.56	145.68	103.73	9.80	10.2
Long-wave A mode (th.)*	7.85	1.469	116.90	167.03	9.58	
// (num.)	//	1.511	118.72	166.39	9.73	

*results given by D. Fabre.

Table 2: Four-vortex modes: linear stability (th.) at $\epsilon = 0.02$.

	Λ	β	θ_o(deg)	θ_i(deg)	$\varrho = \rho_i/\rho_o$	ρ_o/ρ_i %
Most amplified S1 mode (th.)*	1.2566	3.07	82.81	132.53	48.5	2.0
Long-wave S1 mode (th.)*	7.85	1.62	140.36	104.35	10.00	10
Long-wave A mode (th.)*	7.85	1.40	110.13	167.54	9.35	10.7

*results given by D. Fabre.

Missing Slide

Data analysis

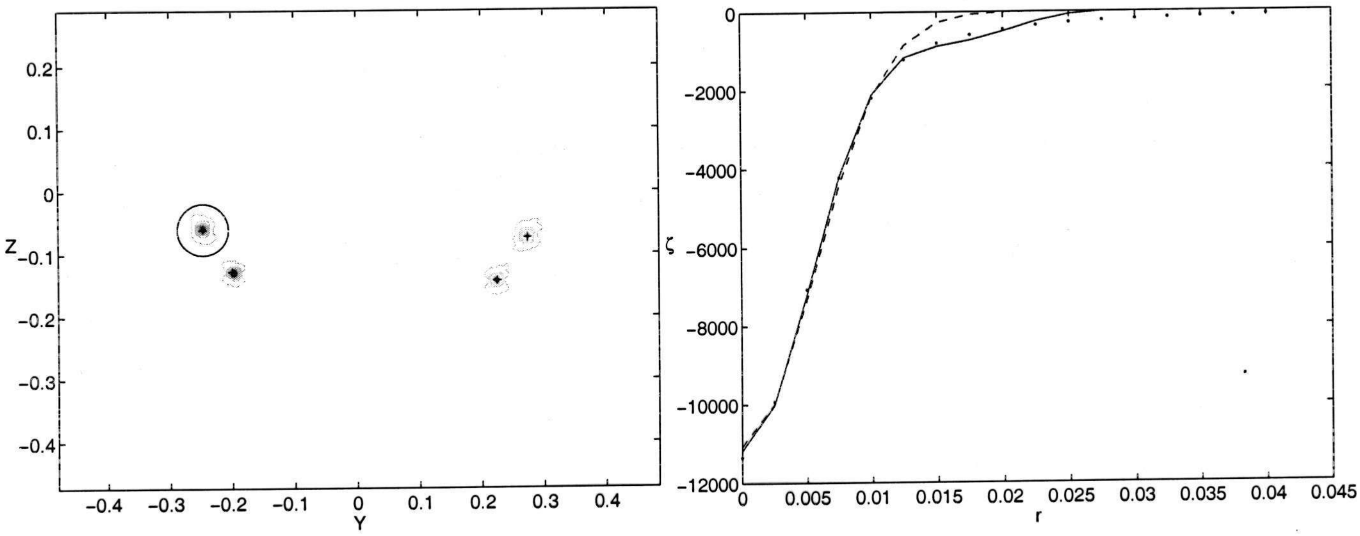


Figure 4: vorticity contour and axisymetric profil of vorticity

Positions and characteristics of vortices

Wing	x/b	y	z	Γ	δ
left	29.83	-0.281	-0.381	-4.107	0.0237
right	29.83	0.245	-0.344	+4.187	0.0236

Physical parameters of the simulations

Γ (m^2/s)	L (m)	ϵ	$\bar{\nu}$ (m^2/s)
$\pm$ 4.15	0.527	0.0460	0.0077

Numerical computation

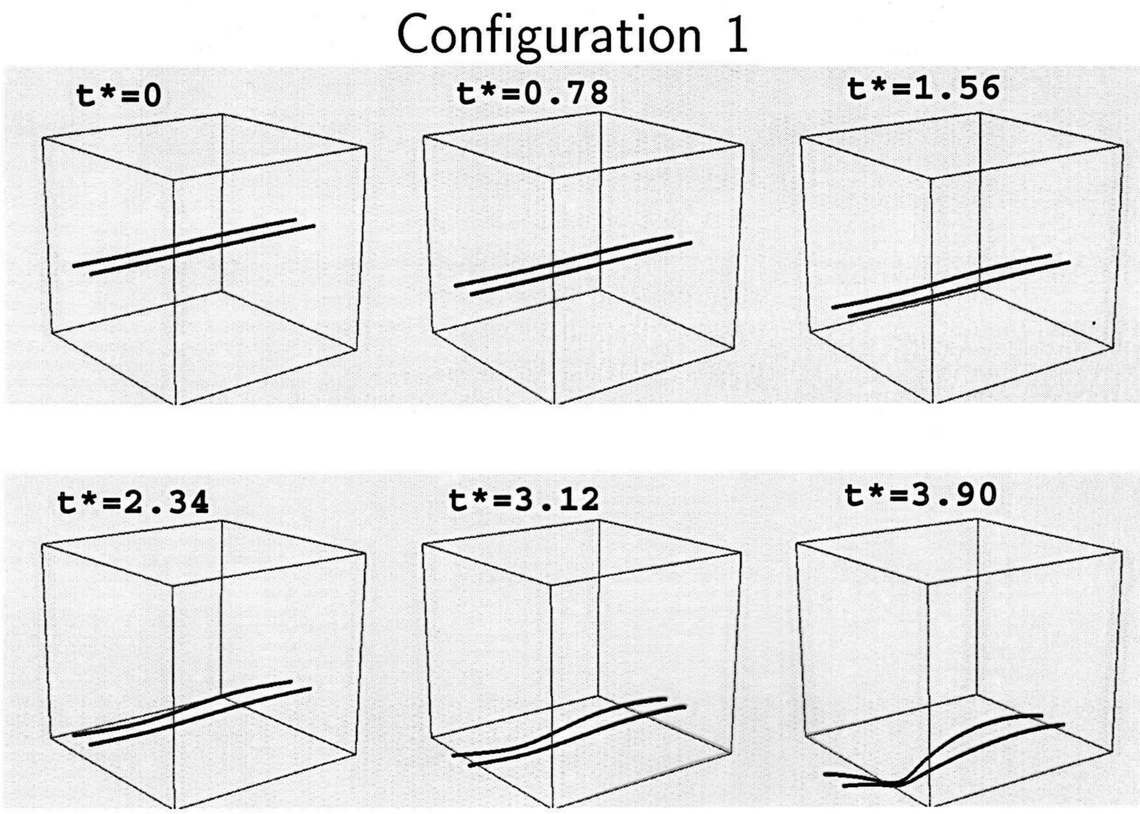


Figure 5: Vortex Filament Simulation of the far wake for configuration 1.

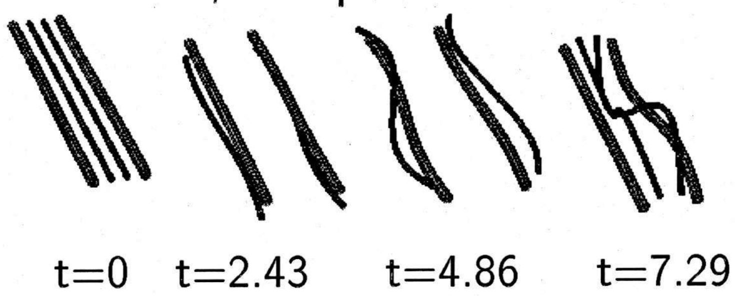
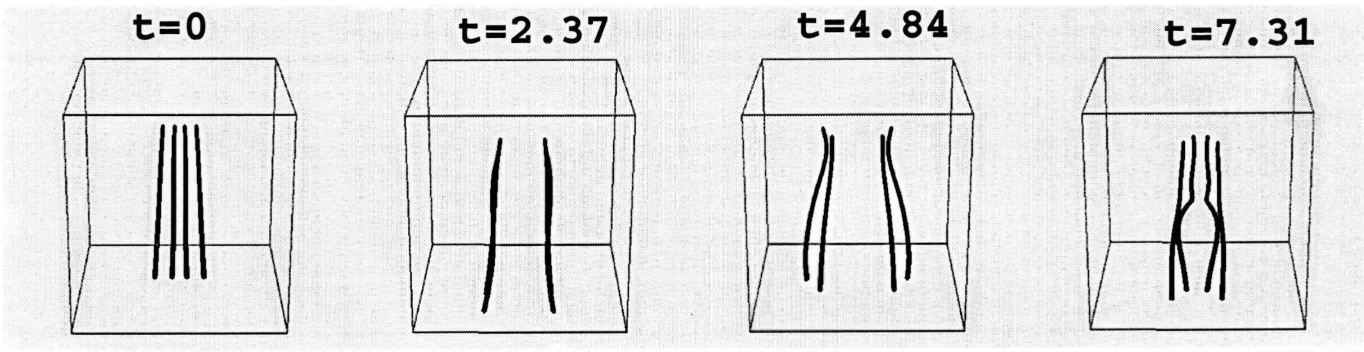
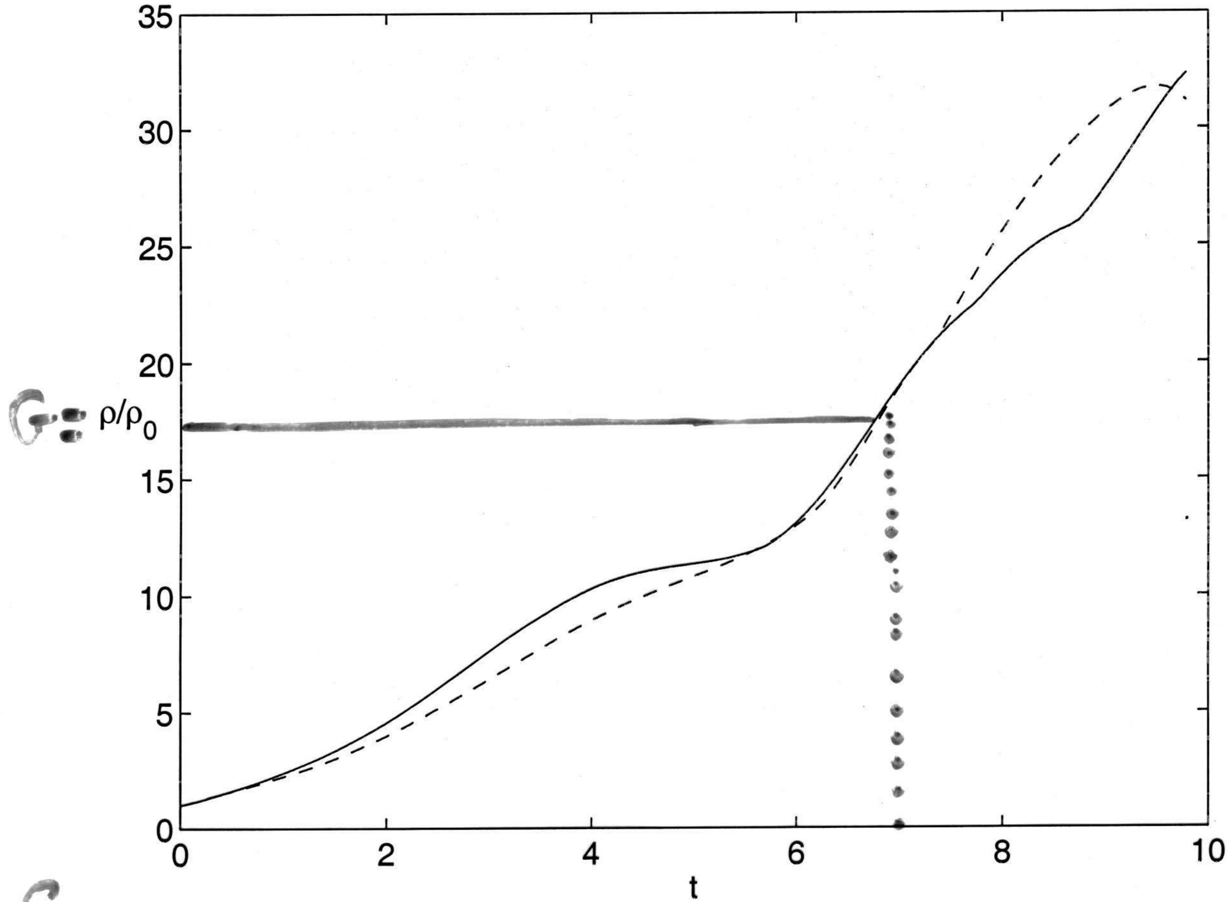
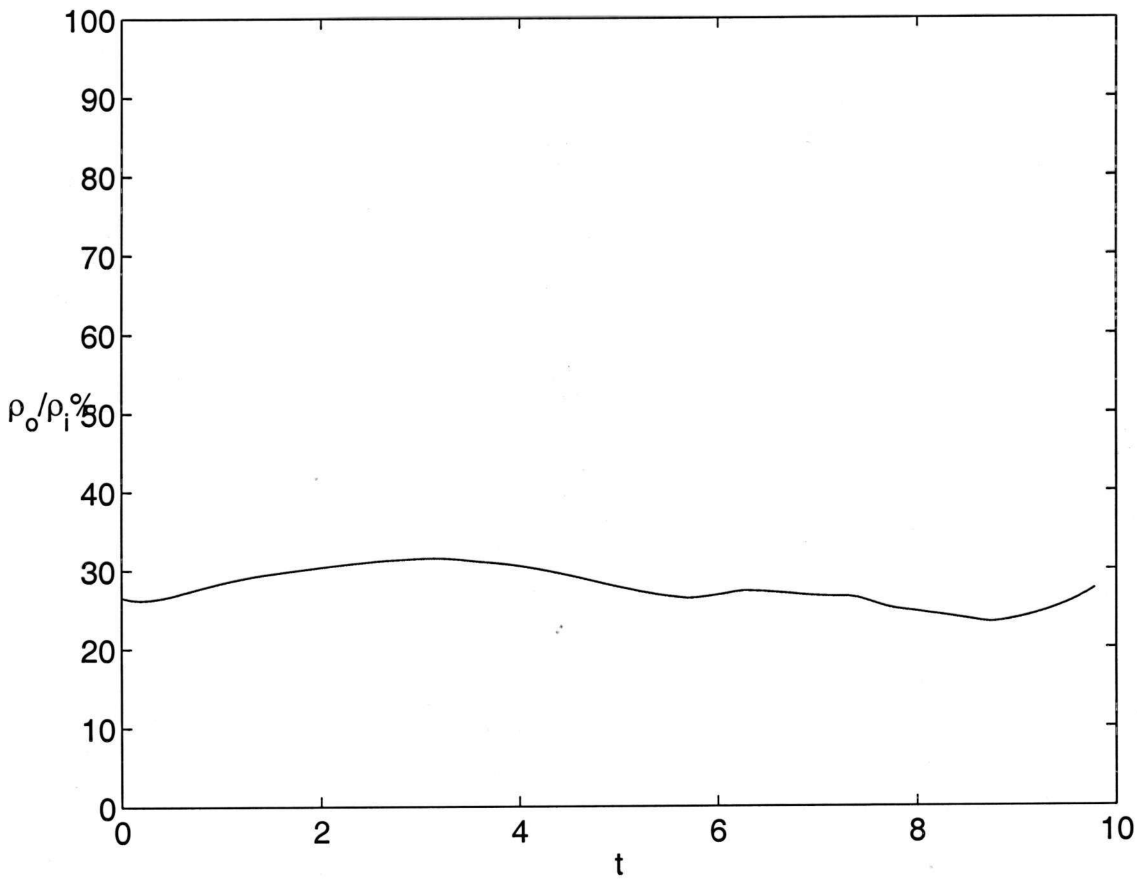


Figure 6: Four vortex wake: $\rho_0 = 0.02$, $\epsilon = 0.1$, $\Lambda = 10.21$, $\Gamma_i/\Gamma_o = -0.3$ and $b_i/b_o = 0.3$





$G_{Fabre \& Jacquin} = 44.3$



ρ_o/ρ_i $_{Fabre \& Jacquin} = 24\%$